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THESIS

GENERAL LOSS FUNCTION APPLIED TO SATELLITE SCHEDULING OPTIMIZATION

by

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September 2014

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Satellite imagery utilization is an oversubscribed problem and therefore requires optimum scheduling methodology to maximize the use of the systems. There are many methods to determine performance of a scheduling algorithm, many of which rely on comparison to already established standards. Based on Taguchi's quality loss function formulation that was developed for the manufacturing industry, four general quality loss functions are presented. These loss functions show the dollars lost when two different performances are changed. The two examined are (1) system response time to user image request and (2) total number of image requests satisfied. The general loss function is applied to the satellite scheduling problem to associate losses captured by the algorithm into a common unit, dollars lost. These loss functions, once developed, help decision makers determine how best to utilize their systems in terms of expected bottom line value to the company.

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GENERAL LOSS FUNCTION APPLIED TO SATELLITE SCHEDULING OPTIMIZATION

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ABSTRACT

Satellite imagery utilization is an oversubscribed problem and therefore requires optimum scheduling methodology to maximize the use of the systems. There are many methods to determine performance of a scheduling algorithm, many of which rely on comparison to already established standards. Based on Taguchi's quality loss function formulation that was developed for the manufacturing industry, four general quality loss functions are presented. These loss functions show the dollars lost when two different performances are changed. The two examined are (1) system response time to user image request and (2) total number of image requests satisfied. The general loss function is applied to the satellite scheduling problem to associate losses captured by the algorithm into a common unit, dollars lost. These loss functions, once developed, help decision makers determine how best to utilize their systems in terms of expected bottom line value to the company.

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EXECUTIVE SUMMARY

Determining how to make decisions and communicating those decisions to the key stakeholders is an important aspect of systems engineering. Given the very complex nature of many of the systems in today's world, having a methodology to compare complex factors in common terms, according to Langford, of general energy (joules or electron volts), matter (in terms of mass, e.g., kilograms or pounds), material wealth (e.g., dollars), or information (bits of data) (EMMI) is a desirable objective (Langford 2012).

Optimally scheduling satellites has been a focused research area since the 1960s when satellites where first used for taking images. Demand for intelligence from satellites has only increased. Therefore, different optimization techniques have been proposed over those 50 years, all trying to accomplish maximum performance of collecting imagery from a satellite constellation.

Determining quality of a function is often referred to as conformance to specifications. Taguchi transformed the understanding of quality in the manufacturing industry by reframing the problem. Instead of looking at quality as a binary good or bad, he posed the construct of a loss function that mapped quality of achieving a target performance (or an event) within an upper and lower variance. He realized that consumers see a performance difference within these specifications the moment they deviate from the optimum target value. He then defined quality loss as the total loss to society for a part that operates outside the limits of its specifications from birth to eventual disposal (Taguchi, Chowdhury, and Wu 2005). One of the main benefits of this methodology is that it relates quality, an abstract idea, to quantifiable terms of dollars lost due to deviation from specifications. This relation makes the decision-making process about whether to invest in better machinery/processes in the factory a straight forward dollar comparison of alternatives and performances. A loss function is a graphical representation of the loss or regret associated with an event.

A general quality loss function, which applies to lifecycle issues (e.g., all phases of the acquisition process), was developed by Langford and Choi. This general quality loss function is applicable to almost all systems once it is developed and it is determined which phase of the acquisition cycle a product is in. The user need only to develop the individual loss function relating to the decision that is being made and can determine the loss in any of the units of general energy (joules or electron volts), matter (e.g., mass in kilograms or pounds), material wealth (e.g., dollars), or information (bits of data) (EMMI) (Choi and Langford 2008).

The general loss function methodology is then applied to the problem of satellite optimization to put different outcomes in terms of losses of EMMI, in this case dollars. The ability to put different performances of the optimization problem in terms of dollars allows satellite operators to better determine satellite schedules for optimum bottom line performance.

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I. INTRODUCTION

A. BACKGROUND

Determining how to make decisions and communicating those decisions to the key stakeholders is a key aspect of systems engineering. Many of the tools and techniques that have been developed in the last 100 years in project management and systems engineering have revolved around making decisions in the best way possible. Given the very complex nature of many of the systems in today’s world, having a methodology to compare complex factors in common terms of general energy (joules or electron volts), mass (kilograms or pounds), material wealth (e.g., dollars), or information (bits of data) (EMMI) is a desirable objective (Langford 2012).

Optimally scheduling satellites has been a focused research area since the 1960s when satellites where first used for taking images. In the 50 years since, the demand for intelligence from satellites has only increased. Satellite scheduling can now be thought of as an oversubscribed problem, meaning in general, there are always far more targets available to take an image of than available satellites or bandwidth to satisfy all the requests (Yao et al. 2010, 10–18). Therefore, a multitude of different optimization techniques have been proposed throughout the years, all trying to accomplish specific goals. Some optimize for the most number of images, some factor in priority of images, try to maximize profit, others the life of the satellite. There is also a push to automate the scheduling problem and make running the calculations as fast as possible, in case changes need to be made.

Determining quality of a system function is often referred to as conformance to the specifications of the performance of that function. Taguchi transformed the understanding of quality in the manufacturing industry by reframing the problem. Instead of looking as quality as a binary good or bad, he realize d that consumers see a performance difference on items the moment they

deviate from the optimum target value, *m*. He then defined quality loss as the total loss to society for a bad part initial concept development and prototyping to eventual disposal (i.e., throughout its lifecycle). Realizing that every single part would have its own, unique quality loss function, Taguchi used a discrete set of quadratic equations to model the quality loss on all parts, making the mathematics easier for the user (Taguchi, Chowdhury, and Wu 2005). This quality loss function, when used correctly, helps manufacturers determine when they should invest in their manufacturing and design process and when it is better not to invest. One of the main benefits of this methodology is that it puts quality, an abstract idea, in terms of dollars lost due to manufacturing variation. This portrayal makes the decision-making process about whether to invest in better machinery/processes in the factory a straight forward dollar comparison.

Langford and Choi took Taguchi's idea of quality loss in the manufacturing field, and applied the idea to weapon system acquisition (Choi and Langford 2008). They developed a general quality loss function, which can be applied to all four stages of a weapons systems lifecycle (concept and technology development, system development and demonstration, production and deployment, sustainment and disposal). This general quality loss function is applicable to all systems that achieve sustainment through competing or opposing uses of resources. This tension creates the forces that signify the tradeoffs required to ultimately satisfy the customers for imagery. The basic physics behind satellite motion and therefore the scheduling of satellite imagery depends on the orbital motion, also known as the two-body problem (Curtis, 2011). In Appendix A, quaternions are introduced as a method to show the fundamental tension within the satellite motion equations and therefore show a loss function is appropriate to describe a performance of the system. The phase of the acquisition cycle in part determines the shape of the quality loss function. The user need only to develop the individual loss function relating to the decision that is being made and then determine the loss in any of the appropriate units EMMI.

B. OBJECTIVES, METHOD, AND RESEARCH QUESTIONS

The objective of this thesis is to determine if the general quality loss function, when applied to the satellite optimization problem, can provide a decision-making tool for satellite developers and operators to optimize imagery collection. From a methodological perspective, a more thorough background in the satellite optimization is presented. Then, a specific satellite optimization is investigated. This optimization was selected because it clearly shows the tension between two important factors in optimization, (1) how quickly do we need to respond to image requests and (2) how many image requests can we accomplish. The rationale for these two factors is they represent the tension by which optimization is achieved. For example, if a user requested 10 different images demanding a very fast response from the satellite system to provide the requested imagery, then most likely the satellite will need to respond in a non-optimal way to accomplish the requests. However, if the response time can be slower, then the satellite can schedule more images in-between those 10 requests and in general take more images.

After the optimization algorithm is discussed, a stakeholder analysis is accomplished. The stakeholder analysis revealed three general classes of stakeholders when looking at how fast image requests need to be accomplished. These three distinct time frames are used to make three different quality loss functions and provide insight into the analysis of the general loss function for use with the satellite optimization problem.

After the stakeholder analysis, Taguchi's loss function is discussed to establish the basis for applying the quality loss function to capture the tension in the scheduling process. Taguchi's loss function is used in the manufacturing industry to determine when making manufacturing and process improvements, for the sake of improving product quality, is beneficial to a company in terms of expected return on investment (2005). Langford and Choi's work on the general quality loss function is presented that builds upon Taguchi's concept and applies it more broadly to the acquisition lifecycle. An abbreviated proof of Langford and

Choi’s work in developing the general loss function is presented in the text as well as an example application of the general loss function (2012).

Finally, the general loss function is applied to the satellite optimization problem in order to show the different tradeoffs in performance of the optimization algorithm in terms of loss of dollars. Stating overall quality (in Taguchi’s case) or number of images taken versus how fast image requests must be met in terms of dollars helps decision makers think objectively about abstract ideas. Therefore, for the application of the satellite optimization problem, four different loss functions are developed, and three different optimal loss function curves are generated for three different general use cases. General loss curves are then shown to apply to the original optimization algorithm and suggestions on how these functions would be used by developer and operators in the satellite industry are provided.

II. SATELLITE OPTIMIZATION ALGORITHM

A. SATELLITE SCHEDULING

Earth observation satellites are satellites that use optical or other remote sensing payloads to observe and generate images of ground targets (Yao et al. 2010, 10–18). Since the time satellites first started being used for military and civilian applications, there has been interest in taking images of objects on the ground. The first successful launch of remote sensing satellite was part of the Corona mission, which successfully launched August 10, 1960 (Olsen 2007). This satellite accomplished image taking via traditional film; then it dropped film cartridges at specified times and was collected on the surface of the earth. Since that time, the capabilities of remote sensing satellites have increased in every way. Today's most advanced imaging satellites, such as WorldView-2, can view the earth at sub .5 meter resolution in panchromatic (black and white) and has a resolution of 1.85 m multispectral (Digital Globe 2014). They also use digital imagery and send all saved image information to the ground via radio frequency communication with ground stations.

The demand for satellite images is vast, spanning the industries of meteorology, oceanography, fishing, agriculture, biodiversity conservation, forestry, landscape, geology, cartography, regional planning, education, intelligence, warfare, and remote sensing research, with the desired targets to be imaged above that of the available resources (Luccio 2012; Stottler 2010; Olsen 2007). From the year 2008 to 2013, the total space revenue from remote sensing has grown from $700 million to $1.5 billion dollars as the demand for images continues to increase in both government and industry markets (Satellite Industry Association 2014). In these situations, the field of operations research is valuable in trying to optimally utilize such satellites to get the most targets imaged as possible. In order to understand the problem better, Figure 1 is introduced. Figure 1 is a graphical representation of the satellite imaging process (Yao et al. 2010, 10–18). Since satellites are restricted in their orbits and are moving very

fast (about 8 km/s in low earth orbit) in relation to the ground, a satellite will pass a ground target for only a limited time. Depending on target size, this could be a fraction of second that the satellite is over the target. The entire imaging capture process must then be accomplished in that limited time window when the target is within range of the satellite imaging sensor. For targets that are not directly under the satellite, most sensing satellites can perform roll operations to rotate the sensor or satellite so that the target is imaged. For targets that are close to each other but require rolling movements, calculations must be performed to ensure the satellite will be able to roll and stabilize before the imaging process needs to start. Figure 1 shows a satellite with three targets to image, and targets B and C can be seen to require rolling operations to image.

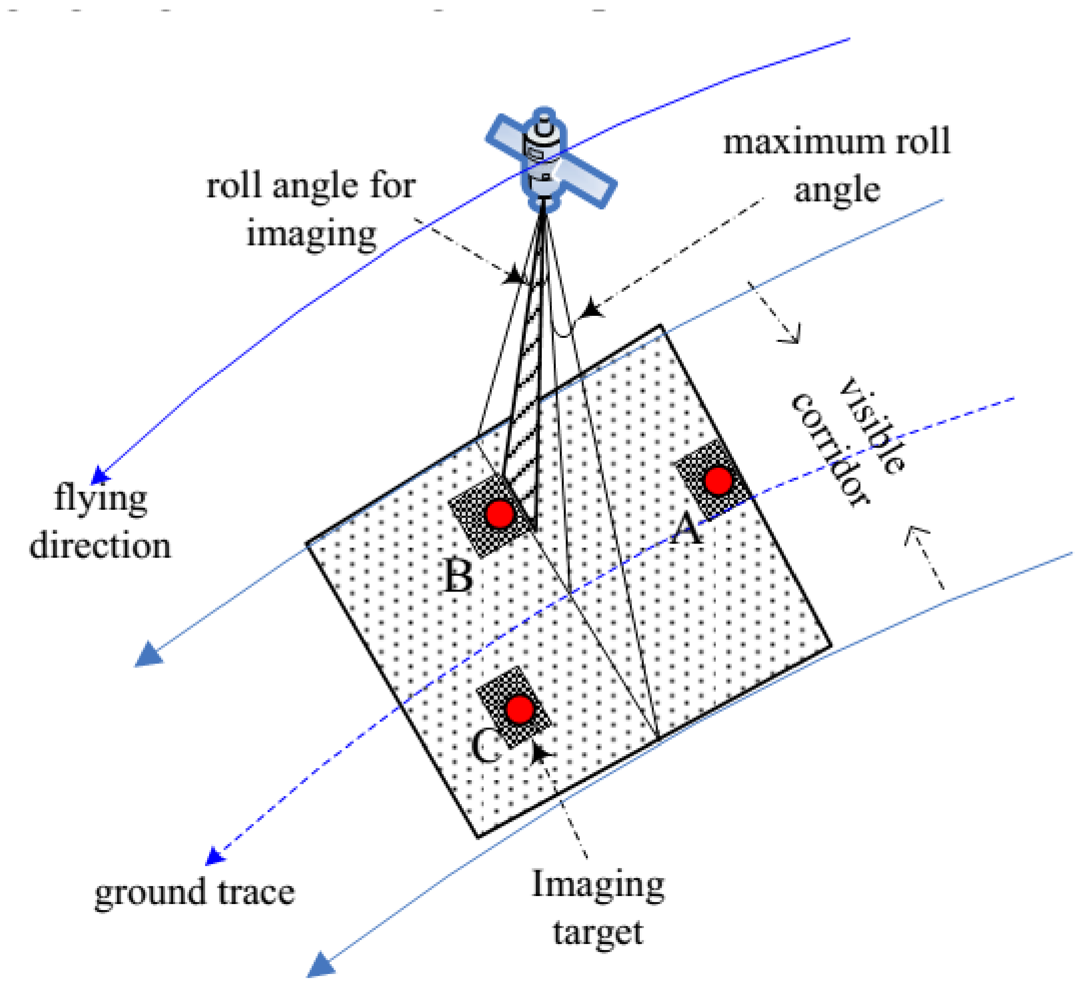

Figure 1. Satellite Image Process (from Yao et al. 2010, 10)

Given the above definition of the satellite imaging process, the satellite scheduling problem can be viewed as a single machine scheduling problem with a time window constraint and setup time correlative with the order that targets are to be imaged (Yao et al. 2010, 10–18). The problem of scheduling a satellite was also determined to be NP-hard, which means the satellite scheduling problem is at minimum as hard as solving a non-deterministic polynomial time (Barbulescu et al. 2004, 7–34). With the increase of satellites in orbit and constellations of satellites being developed, the multi-satellite imaging scheduling problem has become a research priority in recent years. There have been heuristic algorithm approaches (De Florio 2006; Pemberton and Galiber 2001, 101–114), meta-heuristics such as evolutionary algorithms (Globus et al. 2003),

artificial intelligence techniques (Stottler 2010), and decomposition based algorithms (Yao et al. 2010, 10–18). There is also a focus to schedule satellites dynamically giving them onboard decision-making capability so that if there are environmental factors that would inhibit a particular imaging event (e.g., clouds, smoke, or high atmospheric interference) the satellite computers autonomously choose a different mission to be accomplished (Pemberton and Greenwald 2002, 165–171).

Given the amount of research already accomplished in the field of satellite constellation optimization, there are many different algorithms available to select in order to solve the problem. Some offer increased computational efficiency over others, but with the speed of computers increasing, such gains are becoming less important. For the purposes of this research, an algorithm that shows the tension between the time available or period and the number of operations to make is desirable. When there is a tension between two items, it leads to a negotiation between the two factors. Chapter IV will explore the quality loss functions role in determining how to model and predict results based on the negotiation between two factors.

B. SELECTED OPTIMIZATION ALGORITHM

In order to frame the satellite optimizations problem, parameters must be established. The following section is an explanation of Sergio De Florio's heuristic based optimization algorithm that shows the desired tension between two factors (De Florio 2006). The systems being considered are two or more satellites in low or medium earth orbit, one or more ground station and a list of targets to be observed. Each satellite will have its own operability and structural limitations. The primary constraints for optimizing the system are derived from user requests. There are also constraints based on the ground stations, targets, system limitations, time constraints, and on-board resources.

1. System Constraints

Since operations planning and scheduling is primarily constrained by user requests and mission systems needs typical constraints are included in the algorithm. These constraints include:

1. final product commissioner—method of downloading data
2. target locations on earth
3. target dimension and shape
4. target acquisition time—day or night capture
5. image resolution—requested by user, depends on satellite capability
6. type of imaging sensor
7. type of data—if image taking device has different modes
8. number of images on a target—one, multiple, or periodically
9. spacecraft azimuth—if user requires spacecraft to be moving in a certain direction for data take
10. spacecraft elevation angle in relation to the target
11. allowed time window of image request
12. required response time for image delivery
13. type of priority

The next class of constraints is dependent on the Satellites themselves and include:

1. satellite orbit—precise prediction on satellite orbits are completed via simulation software on the ground
2. power storage—how much power is available and how fast it charges is modeled for each satellite
3. power consumption—running tally of all operations must be maintained to ensure there is enough power over the scheduling horizon
4. data storage—ensure on-board storage has enough memory to store new images without overwriting old ones
5. payload—type of sensor and its field of view characteristics
6. data download—rates for different operations are characterized
7. inter-satellite links—if satellite has capability to send information satellite-to-satellite instead of only to ground station

For ground stations, both the visibility horizon (how long the satellite will be in view of the ground station to communicate) and the handshake time (how long it takes to communicate information) is taken into account. Targets are modeled as closed contour regions with a certain location on the Earth's surface and defined by a series of points. System limitations and constraints can be summarized as time constraints and on-board resource limitations. The following time constraints are considered:

1. assumption that spacecraft can only do one operation at a time
2. spacecraft revisit limitations on targets—Number of opportunities satellite will be within view of the target in the scheduling period
3. ground station contacts—number of available ground station contacts predicted over the scheduling period
4. attitude maneuvers—time required to adjust to the predicted attitude of target for data take
5. payload management—time required to manage the payloads (power on/off, process images, or run heaters)

The on-board resource limitations considered are:

1. on-board power availability—continuously monitored during the scheduling to ensure enough power is available
2. limited on-board data-storage—continuously monitored during scheduling to ensure enough storage available
3. sensor operability—minimum and maximum elevation angles to target are considered
4. data-download rate—parameter for each satellite and ground station pair defining their data transfer rate

2. Types of Missions

After the initial constraints are complete, it helps in the optimization algorithm to define a set of standard missions or operations that the satellites will be commanded to complete. For this application, the following operations are defined: monitoring pass, download pass, image-take, on-line Image-take, inter-satellite link, and system maintenance. Based on the missions and the constraints, a set of temporal reasoning constraints are established (De Florio 2006).

1. An operation is allowed to be commanded by a ground station when the time constraints of that operation fall within an allowed time window when other operations are happening.
2. Image-takes, which occur at the same time as ground station contact, are grouped together and called an on-line image-take.
3. An inter-satellite link is allowed if the satellite is within range of a separate satellite, which is in contact with a ground station.
4. For each inter-satellite link, the other linked satellite connected to the ground station is associated with the image taking satellite for that time window.

3. Assignment of Priorities

At this point in the formulation of the optimization, the method of assigning priorities is defined. Priority is a value [high (urgent), medium, and low] that helps to properly assign image take operations in the order most desired by the user and by the operation of satellite itself. All the image take operations are grouped first by their respective type then are sorted in the list by their priorities. The first priority for assigning location in the queue for each image is the user priority, which is assigned by the user for each task (high, medium, and low). Next the operation, given a set of equally prioritized operations by the user, sorts the list based on type of operations. Ground stations contacts are set to the highest priority because these operations by definition cannot be missed unless a direct command from the user allows it. Next, online image-takes are prioritized, followed by inter-satellite link, and lastly ordinary image-takes are sorted.

Once the lists are sorted by type and priority, the initial scheduler is run and all the satellite ground station passes are first assigned, which means the first operations plan will only contain satellite monitoring passes (no image takes have been scheduled). Next the scheduler goes through chronologically sorted potential task lists for each satellites sequentially. At each step in the schedule, the schedule attempts to insert an operation into the schedule that is allowable according to the defined constraints above (De Florio 2006). The user priority is first considered, and if two items are of the same priority, than the lists are scheduled in order by: on-line image-takes, inter-satellite link contact, and image-

takes. As each task is added into the schedule for each satellite, various checks are done to ensure the task is allowed. These include, time checks, spacecraft status (battery, storage, or position), and the next opportunity to download image is scheduled.

Since the user defined priority determines where in the queue of operations an image take will happen, that is the key item that determines the level of performance of the optimization algorithm. Given that, the figure of merit that determines the priority is very important. For this optimization, two figures of merit were chosen, (1) System response time and (2) number of images. The system response time is defined as the amount of time from when an image is requested until the image is ready to use. The number of images is the satisfied number of requests that were accomplished in the scheduling period. Therefore the basic priority assessment rule can be defined as Equation (1), based on information from www.stsci.edu:

$$T = \alpha * \mathrm{I} + \beta * \mathrm{S} \tag{1}$$

where *T* is the priority value, *I* is the number of image taking opportunities, *S* is the time slot and $\alpha, \beta \in \mathbb{R}^{+}$ are respectively the weights of *I* and *S*. The highest priority possible is one and the goal is to find the values of α and β to maximize the number of satisfied image requests or to minimize the response time. Therefore, it can be shown there is a two dimensional space $\alpha - \beta$ that must be explored (Figure 2).

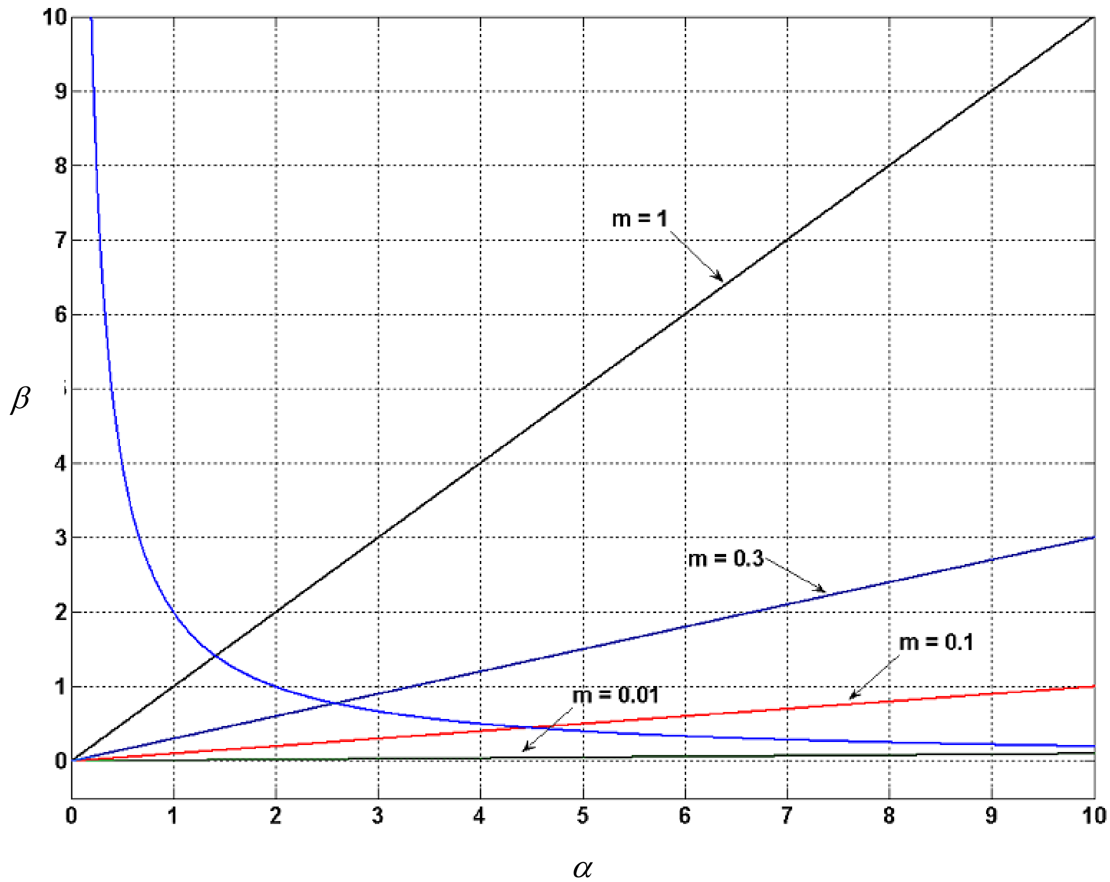

Figure 2. Space $\alpha - \beta$ (from De Florio 2006, 4)

The two curves that define Figure 2 are relations between α and β, which are Equations (2) and (3) respectively:

$$\alpha = m * \beta \tag{2}$$

$$\alpha * \beta = \cos(t) \tag{3}$$

The order that the operations are listed in the queue to be processed depends on the value of the priorities. Therefore, from the simple form of Equation 1 it can be inferred that with a chosen value of *m* and $\frac{\alpha}{\beta}$ remaining constant, the priorities assignment order does not change (De Florio 2006).

4. Results of Original Algorithm

Once the algorithm was developed, De Florio ran some simulations given some assumed system constraints and obtained initial results (De Florio 2006). While testing the performance of the new scheduling algorithm, a first-in-first-out scheduling methodology was used as a comparison. The satellite constellation parameters that were used for calculations are show in Table 1.

Table 1. Satellite Constellation Constraints (from De Florio 2006, 5)

1 ORBITAL PLANE CONFIGURATION				
Satellite	SAT1	SAT2	SAT3	SAT4
a (km)	6893.14	6893.14	6893.14	6893.14
e	0	0	0	0
i (deg)	97.4	97.4	97.4	97.4
RAAN (deg)	0	0	0	0
ω (deg)	0	0	0	0
TA (deg)	0	90	180	270
2 ORBITAL PLANES CONFIGURATION				
Satellite	SAT1	SAT2	SAT3	SAT4
a (km)	6893.14	6893.14	6893.14	6893.14
e	0	0	0	0
i (deg)	97.4	97.4	97.4	97.4
RAAN (deg)	0	0	120	120
ω (deg)	0	0	0	0
TA (deg)	0	60	300	360
a = semi-major axis, e = eccentricity, i = inclination TA = true anomaly, ω = argument of perigee RAAN = right ascension of ascending node				

Once the satellite constraints were loaded into the model, it was assumed that only one ground station and one user will be utilizing the system. There were 620 targets distributed over one week of time and each target was a 7km x 7 km square surfaces. Also, there were three simplifying assumptions made for the simulation results: (1) all image-takes were performed in one mode, (2) all user priorities were the same, and (3) no deadline was assigned to any of the requests. In addition to these assumptions, the model itself makes the assumption that each satellite can only do one task at a time. This assumption is slightly misleading, since some of the single tasks are actually multiple operations, like an on-online image take where the satellite is taking images and

transmitting them at the same time. Once all constraints were considered and lists of different missions, target visibility time windows, ground stations time windows, satellite power, and satellite storage requests were generated, then the priority lists were made. These lists were made multiple times, varying the value of *m* in Equation 2, trying to find an optimal value for *m* in terms of maximizing the two selected performance metrics, system response time and total number of image requests fulfilled. In Figure 3, De Florio shows the two metrics and their respective performance as *m* is varied from zero to 10. An example of the results found by De Florio are shown in Figure 3.

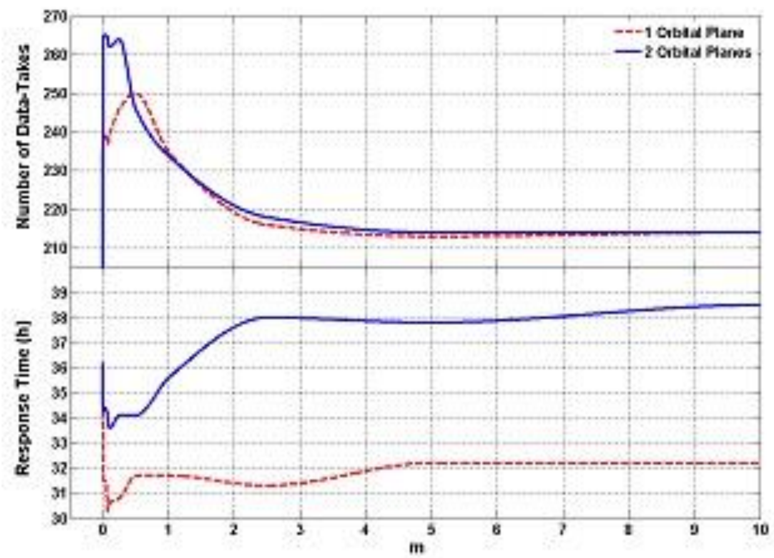

Figure 3. Variation of Number of Satisfied Requests and System Response Time with *m* (from De Florio 2006, 5)

In doing this initial searching for the optimal value of *m*, De Florio (2006, 6) made the following three key insights:

1. For each configuration there are two values of *m* corresponding to optimal values of both the figures of merit.
2. For each figure of merit the variation with *m* is similar for the two different configurations.
3. The values of *m* that respectively maximize the number of satisfied request and minimize the response time, are akin.

While the first two of these insights regarding *m* are fairly straight forward, the last item is surprising. Given that the two figures of merit are in tension with one another, why would the values of *m,* which optimize each of them, be similar? The answer is found in his simplifying assumptions. De Florio assumed that all image requests were of the same priority and had no time constraints as to when they had to be completed (2006). Since the number of high priority images is not taken into account, the proposed optimization algorithm results have limited value to decision makers who are trying to determine if it is better to prioritize the response time of the image requests or how many image requests are fulfilled. Therefore, a general loss function was developed, which assists that decision by making it easier to determine, in terms of dollars, the optimal system response time and number of image requests to fulfill.

III. STAKEHOLDER ANALYSIS

A. DEFINITION OF STAKEHOLDER

> The word ‘stakeholder,’ one who has a stake in the outcome, is most typically an entity (a person either acting alone or representing an organization) who can influence the conceptualization or funding of the development project, or the product’s or service’s acceptance, operations, or disposal. (Langford 2012)

Stakeholders are those who affect or are affected by the “decision-making activity” that “influences the product or service. In a broader sense, it is someone with an” interest “or concern, and specifically someone at risk due to the product or service” (Langford 2012). Stakeholders can also be defined by industry, which the International Council of Systems Engineers (INCOSE) does by defining stakeholders as: “A party having a right, share or claim in a system or in its possession of characteristics that meet the party’s needs and expectations” (ISO/IEC 2008).

Looking at a broad scope of definitions of stakeholder, Langford notes there are common themes running throughout all the definitions. These themes are in fact so broad that in many cases it is better to ask “Who isn’t a stakeholder?” instead of “Who are the stakeholders?” These common themes include:

1. The stake holder has an interest in the system under development.
2. The stakeholder can provide some insight into the system under development.
3. The stakeholder can influence the development of the system.
4. The stakeholder has an interest in the outcome of the system under development (Choi and Langford 2008).

B. STAKEHOLDER ANALYSIS

Stakeholder analysis is a most important step in the overarching systems engineering methodology because it is vital to first determine the “needs” that

must be satisfied that then are winnowed down to the essential requirements that drive the development of solution(s). All projects will have a minimum number of stakeholders to include the direct customer (the one who pays) and end user of the product being developed. However, there are often unstated stakeholders who have essential requirements for the project. If these stakeholder's needs are not identified early or not met at all, the cost for development and sustainment will increase in later upgrades or may indeed cause the project to fail altogether. Therefore, an effective and efficient process of stakeholder analysis is important to implement. Following a general five step guideline, with each of the five steps having multiple sub-steps, the proper set of functions can be turned into the work-breakdown structure, the appropriate processes can be enacted, and the relevant requirements can be defined. The overarching framework for stakeholder analysis is: "(1) identification of stakeholders; (2) classification of potential stakeholders; (3) determination of potential stakeholder and system relationships; (4) determination of key system stakeholders; and (5) definition of stakeholder requirements" (Langford 2012).

1. Identification of Potential Stakeholders

Identification of potential stakeholder is the first step. As stated before, in many cases the better question to ask, especially for very important assets like satellites, is "Who isn't a stakeholder?" The initial identification of stakeholders should very naturally start with the image needs of the target user(s) of a system and the direct customer(s). In almost all new projects, there will be at least one driving force behind the project that will be an obvious stakeholder. After this first level of identification is complete, the next step is to create scenarios of potential stakeholder interactions. These interactions should relate to every aspect of the system, how it will be used, the timing, fitment, size, reliability, and cost. Asking "What if" questions is also useful at this stage of the brainstorming effort, trying tease out any stakeholders missed in the initial round. At this point, a master list of stakeholders is created, ready for classification (Langford 2012).

2. Classification of Potential Stakeholders

Classification of potential stakeholders is the second high level step of the stakeholder analysis and follows the following four step process: "(1) determination of the system boundaries, (2) classification of potential internal stakeholders, (3) classification of potential first-order stakeholders, and (4) classification of potential second-order stakeholders" (Langford 2012). The first step is to define the system boundaries (including physical, functional, and behavioral boundaries), which is acknowledged to change over the lifecycle of a system. The previously defined use cases and stakeholder interaction scenarios will help to define the system boundaries over the lifecycle of a product. Once the physical boundary of a system is established, by definition, there are then functions of the system inside and outside of the system boundaries. The second step of classification of the stakeholders is to determine all the potential internal stakeholders who are affected by either internal or external functions. Internal stakeholders are those that only interact with internal system elements or with other stakeholders that are classified as internal stakeholders. Thirdly, the set of first-order stakeholders is established. First-order stakeholders are those who are in direct contact with the system, but do not have direction interaction with the internal stakeholders. Finally, second-order stakeholders are defined, which are those stakeholders that are connected indirectly to the system via interaction with first-order stakeholders. The combination of the first and second-order stakeholders are classified as the boundary stakeholders because they interact with external entities across the system boundary. Thus, classification of system stakeholders is complete with a list of internal and external stakeholders (Langford 2012).

3. Determining Relationships between Potential Stakeholders and the System

The third step in the stakeholder analysis is to determine the relationship between the potential stakeholder and the system that is used to prioritize the stakeholders. Prioritizing the stakeholders is important as it facilitates an early

determination as to the key stakeholder needs, likely problems that can be solved and requirements that should be considered while developing the functional analysis of the system. This step is accomplished by grouping stakeholders into their respective system roles, which then helps to prioritize and choose the appropriate stakeholder inputs (Choi and Langford 2008).

4. Identify Key System Stakeholders

The fourth step in stakeholder analysis is to identify key system stakeholders. Key system stakeholders are ranked based on their overall influence and importance for the system across the lifecycle. At various phases in the lifecycle, some stakeholders have greater importance to the eventual outcomes and then other stakeholders. It is incumbent on the elicitation process to determine those significant impacts early in the development cycle, most importantly before design has commenced. After determination of the key stakeholders, three different sets of stakeholders will be established: primary, secondary, and tertiary. Primary stakeholders are defined by those with direct input into the development of the system's functional analysis and in determining how the system will be measured for effectiveness. Secondary stakeholders have limited weighting in the development of the functional analysis of the system and in determining how the system will be measured for effectiveness. However, secondary stakeholders will be incorporated into the system whenever possible within the system boundaries. Lastly, tertiary stakeholders are established and are not considered in the functional analysis or the measures of effectiveness.

5. Establish Stakeholder Requirements

Lastly in the stakeholder analysis, the stakeholder requirements are established. Defining the stakeholder requirements follows a three step methodology as follows: (1) determine the problem, (2) define the need, and (3) define the requirements. A stakeholder problem occurs "Whenever there is a difference between what can be done and what you want to do, and you do not

know how to achieve the desire…" (Langford 2012). For each stakeholder problem statement, stakeholder needs can be found. A need is defined as "…a condition faced by the stakeholder that requires a solution to alleviate it" (Langford 2012). Finally, stakeholder requirements are derived from the needs and form the requirements of the development or use of the final system.

C. IMAGING SATELLITE STAKEHOLDER ANALYSIS

The above stakeholder analysis process is a key piece of the systems engineering model. For the sake of this thesis however, it is assumed that the systems being used are already developed and operational. Since there are already multiple constellations of government and commercial imaging satellites, a stakeholder analysis for the development of a new satellite is outside of the scope of this research. However, determining a set of key users for satellite imagery is appropriate and useful. Specifically, finding users of satellite imagery and grouping them according to their required system response time is desired. Getting a range of users is desired because one of the two parameters of the optimization algorithm discussed above is system response time. In order to develop a loss function for the system response time, a set of user requirements must be determined to establish how fast image requests must be satisfied. After a set of users is established, a determination of different response time ranges is investigated to determine if there are common requirements.

First, an unsorted, incomplete list of users of satellite imagery is established. Common potential uses of imagery are in the fields of meteorology, oceanography, fishing, agriculture, biodiversity conservation, forestry, landscape, geology, cartography, regional planning, education, intelligence, warfare, and remote sensing research (Luccio 2012; Olsen 2007; Stottler 2010). It can be seen in Figure 4 that in these various fields, there is a high disparity of how quickly new images need to be available to be utilized. For applications where there are fast changes happening, such as warfare, forest fires, and severe weather, images over 24–48 hours old will have little to no utility. However, in the

opposite extreme, items like cartography, regional planning, and landscape monitoring might only require images to be updated on a bi-annual basis or longer. There is also a third general group of users who require images within a two- or three-week window. These would be users monitoring tidal action, algal blossoms, ice flows, fisheries, and agriculture.

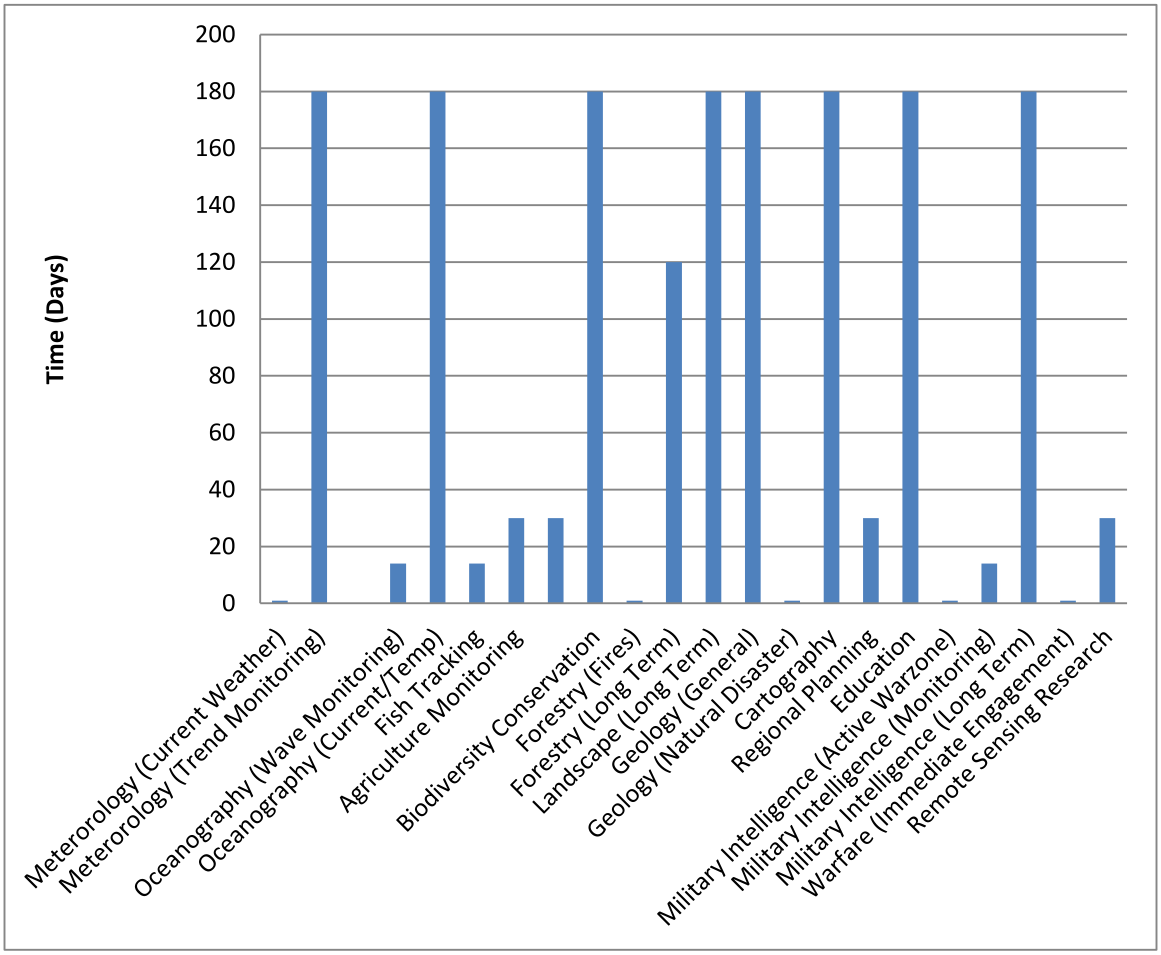

Figure 4. Stakeholder Image Request Time Requirements

Therefore, three distinct requirements for image response time are established. First are users requiring as close to real time imagery as possible, with a target response time of 24 hours. Second are users requiring images to be updated within a two- to three-week time window. Third are those users requiring very infrequent updates, on the order of six months or greater.

IV. QUALITY LOSS

A. TAGUHCI QUALITY LOSS

Quality is thought of in many ways, but for the sake of this paper, the definition will be narrowed to be only that which is measurable through an association with a function. By defining quality in this sense, one can interpret quality to be a property of a function. This means that quality is then deemed to be conformance to performance(s) for that function as required through a set of specifications. Functions have a performance and each performance has a quality. Therefore, each function can be completely defined by the sum of their performances and qualities. Generally speaking, when goods or services are within the specification, they are considered to be of high quality; conversely when goods or services are outside of their specification, they are considered low quality. To be within a specification means all the pieces where built to within the drawing tolerances and assembly of the product was accomplished in such a way that the product performs as intended. When products are built outside of the specification, it can lead to less reliability or in many cases a defective product straight off the assembly line. Quality is an important characteristic to have, as one industry saying says: “the quality is remembered long after the price is forgotten.” (Langford 2012)

In the 1980s, a Japanese engineer and statistician named Genichi Taguchi proposed a different way to view quality. Taguchi proposed viewing quality as it relates to cost and therefore it could be measured in exclusively monetary terms. This loss accrues not only to the designer, developer, and manufacturer but also to the customer, user, and broadly to society as a whole. If one where to abstract and aggregate, this could be viewed as a seller in a buy-sell relationship (Langford 2012). This change of viewing the abstract idea of quality of a product into that of monetary terms was a paradigm shift in the decision-making process for managers of a factory. Now, if Taguchi’s methodology worked, managers could determine real cost benefits analysis on

manufacturing facility and process improvements to determine when changes would pay off (2012). This is a key benefit to the systems engineer who needs to convince management that quality improvements will be cost effective for the company.

Taguchi also changed the view of quality that assumed that if a product were anywhere inside the specifications, it was a uniformly good quality product (2012). Instead, Taguchi believed where the customer became increasingly dissatisfied with the product, the further it got away from the precise target specification. Taguchi realized that for each quality characteristic, there exists a unique function that defines the relationship between economic loss and the deviation of the quality characteristic from its target value. However, since would take too much time and effort to find these unique functions for each quality characteristic, Taguchi found that a quadratic representation of the quality loss function to be efficient and easy (2012). This simplified the mathematics and time required to find countless unique loss functions but provided better approximations to represent a customer's dissatisfaction with a product's performance compared to the traditional binomial good/bad quality determination. The quadratic curve is centered on the target value, which provides the optimum performance in the eyes of the customer. It should be noted that identifying the target value is not easy, and at times a designer must make a best guess (Langford 2012).

There are three general types of quality loss functions: nominal-the-best, smaller-the-better, and larger-the-better (Taguchi, Chowdhury, and Wu 2005). Nominal-the-best characteristic is the type where a finite target value is desired. For nominal-the-best, there are usually lower and upper specification limits. Examples in manufacturing of products that are nominal-the-best are: output of a voltage from a battery, dimensions of a part (length, thickness, weight, density, and height).

Smaller-the-better characteristics are the type where a minimum target is desired, the ideal being zero. For smaller-the-better, there is not normally a lower

specification boundary, but often there will be an upper boundary. Examples in manufacturing of products that are smaller-the-better are: wear on a component, electrical noise in a circuit, or heat loss.

Larger-the-better characteristics are the type where a maximum target is desired, the ideal value being infinity. Conversely to smaller-the-better, there is usually no upper bound for larger-the-better, but often one will find a lower boundary specification limit. Examples of larger-the-better in manufacturing are: material strength, horse power from an engine, or storage density of a battery.

These loss functions offer a way to quantify the benefits achieved by reducing variability around the target. It can be used as a tool to help justify decision makers to improve a process that is already meeting specifications.

For a product with a target value of *m* and deviation from *m* Δ_0, from most customers' point of view, $m \pm \Delta_0$ represents the deviation that functional failure of the product or component occurs. When a product is manufactured with its quality at the extremes of $m+\Delta_0$ or $m-\Delta_0$, some countermeasures must be employed by the average customer. The loss function L with a characteristic of nominal-the-best is described as the following Equation 4.

$$L = k(y-m)^2 \quad k = \frac{A_0}{\Delta^2{}_0} \tag{4}$$

where, *k* is a proportionality constant, *y* is the output, *m* is the target value of *y*, Δ_0 is the variation from *m* and A_0 is the cost of the countermeasure.

The loss function for the smaller-the-better case can also be determined, but is slightly different than the nominal-the-best case. For the smaller-the-better, where the target is zero, the loss function is described as shown in Equation 5.

$$L = ky^2 \quad k = \frac{A_0}{{y_0}^2} \tag{5}$$

where, *k* is a proportionality constant, *y* is the output, A_0 is the consumer loss and y_0 is the consumer tolerance.

For larger-the-better case, the target is infinity and the loss can be written as the following Equation 6.

$$L = k\frac{1}{y^2} \quad k = A_0 {y_0}^2 \tag{6}$$

where, *k* is a proportionality constant, *y* is the output, A_0 is the consumer loss and y_0 is the consumer tolerance.

B. MINIMUM QUALITY LOSS

Taguchi's work was primarily focused in the manufacturing industry. However, it is desirous to be able to apply quality loss in other applications. Langford proposes a general quality loss function, which can be used outside of the manufacturing industry for the rest of the lifecycle stages of a product that includes conceptualization, research, development, integration, operations, maintenance, and disposal (Langford 2012). While Taguchi focused primarily on quality as a performance measure, there are at least seven distinct but not mutually exclusive performance measures that include: effectiveness, efficiency, quality, productivity, quality of work life, profitability, and innovation (Langford 2012). Thus, performance can be viewed in different meanings in different situations for different systems. When referring to more general energy (joules or electron volts), mass (kilograms or pounds), material wealth (e.g., dollars), or information (bits of data) (EMMI) performance can be defined as net work accomplished over a time period (*t*). Since Taguchi's general smaller-the-better, larger-the-better, and nominal-the-best characteristics appear in other applications, a general quality loss function is developed for broader application in managing quality characteristics regardless of domain characteristics, inputs and outputs, and irrespective of preference or specifics for any discipline or field (Langford 2012).

Minimum quality loss was developed by Taguchi as a quadratic for quality, but it can be applied more generally in terms of Pareto-efficient negotiations. Pareto-efficient negotiations are "based on the principle that one-sided benefit to

a party to a negotiation results in an inequitable distribution of losses. So from a stakeholders'" perspective, losses are exchanged as benefits and losses based on the negotiated settlement of the specification (Langford 2012). Therefore the "agreement between two stakeholders is defined as the position whereby neither side to a negotiation has an unfair" or disproportionate advantage (Langford). Therefore, "assuming an idealized negotiation," Figure 5 shows "where two parties incur equal losses about a center point target value *m* (Langford)."

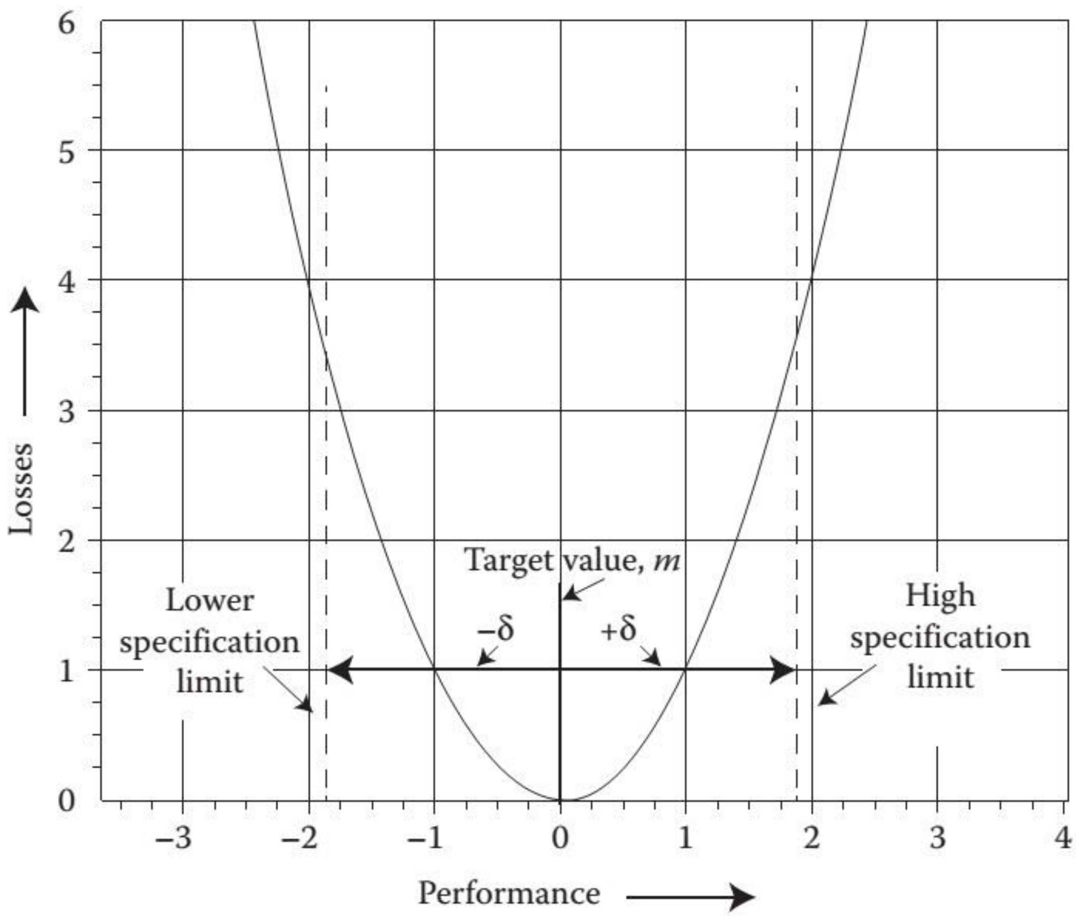

Figure 5. Pareto-efficient Efficient Negotiation (from Langford 2012, 336)

In Figure 5, the minimum loss is defined as the "target value of the critical performance characteristic, *m*," which is negotiated "between two strategies with opposite demands on quality for a given investment" (Langford 2012). In this

example, one party desired that more performance is better, “while the other party considers that smaller” performance is required (Langford). This can be viewed at as a larger-the-better versus smaller-the-better negotiation. Figure 6 shows a plot of the larger-the-better and smaller-the-better strategies plotted separately with the *x*-axis representing performance and the *y*-axis representing losses.

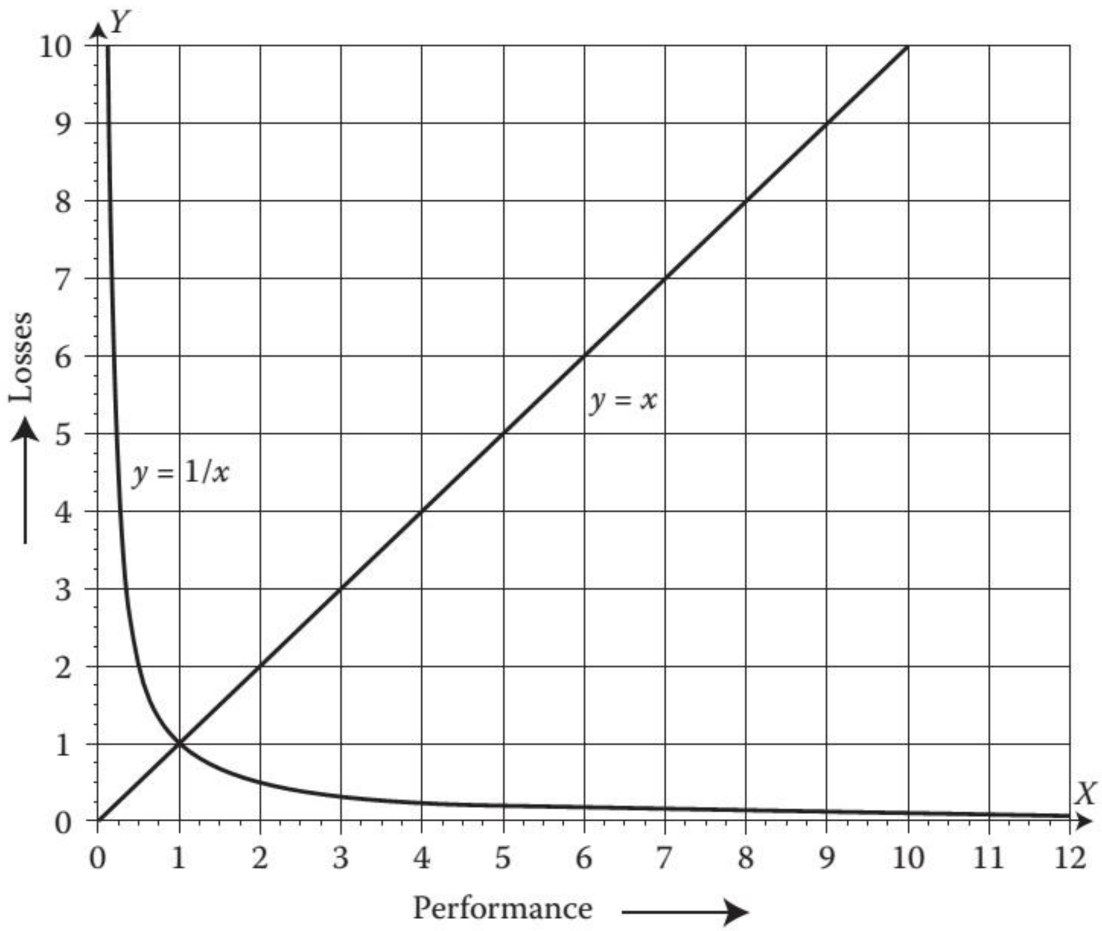

Figure 6. Smaller-the-better (*y*=*x*, Seller) and Larger-the-better (*y* = *1/x*, Buyer) (from Langford 2012, 337)

“Simple addition of the two curves x and $\frac{1}{x}$, results in a pictorial” representation “of negotiation, based on both parties achieving the minimum

loss" (Langford, 2012) Figure 7 "shows the resultant quality loss function" (Langford). The resultant minimum "quality loss distribution has a minimum at $m = 1$, representing the minimum loss that can be caused after the" negotiation (Langford).

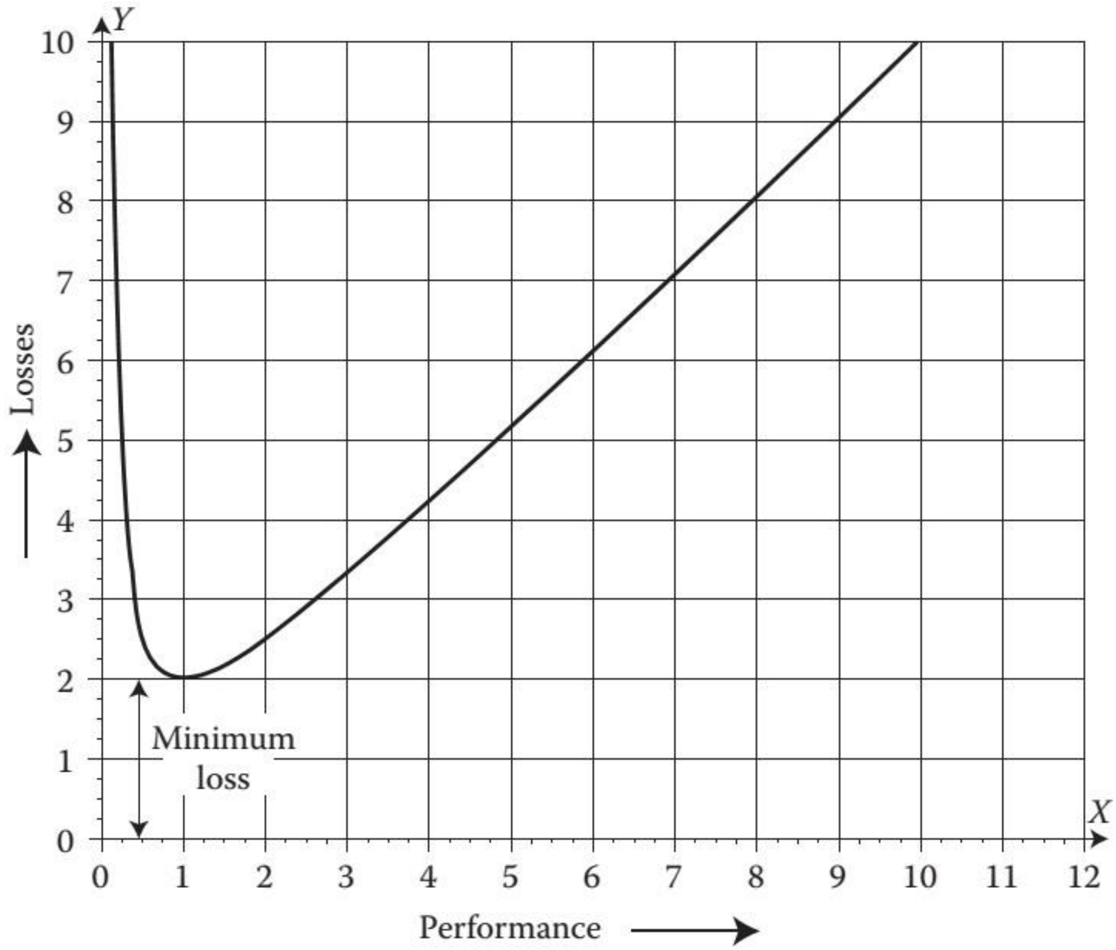

Figure 7. Combing Two Loss Distributions That Compete for a Definitive Target Value, *m* (from Langford 2012, 337)

The minimum loss in Figure 7 represents the most effective performance of the product within the product's specific environment, given the conflicting constraints of smaller-the-better and larger-the-better. The "goal of the negotiations is to minimize the total system losses due to and during" the

lifecycle of the products performance (Langford 2012). It is noteworthy that there are infinitely many “quality loss functions, each with a minimum loss determined by cooperative negotiation between” a “buyer and” a “seller” (Langford). However, “there is only one Pareto-efficient quality loss function that” optimizes “the minimum loss” for any negotiation (Langford). Figure 8 shows an example of different, less effective quality loss functions for a given performance.

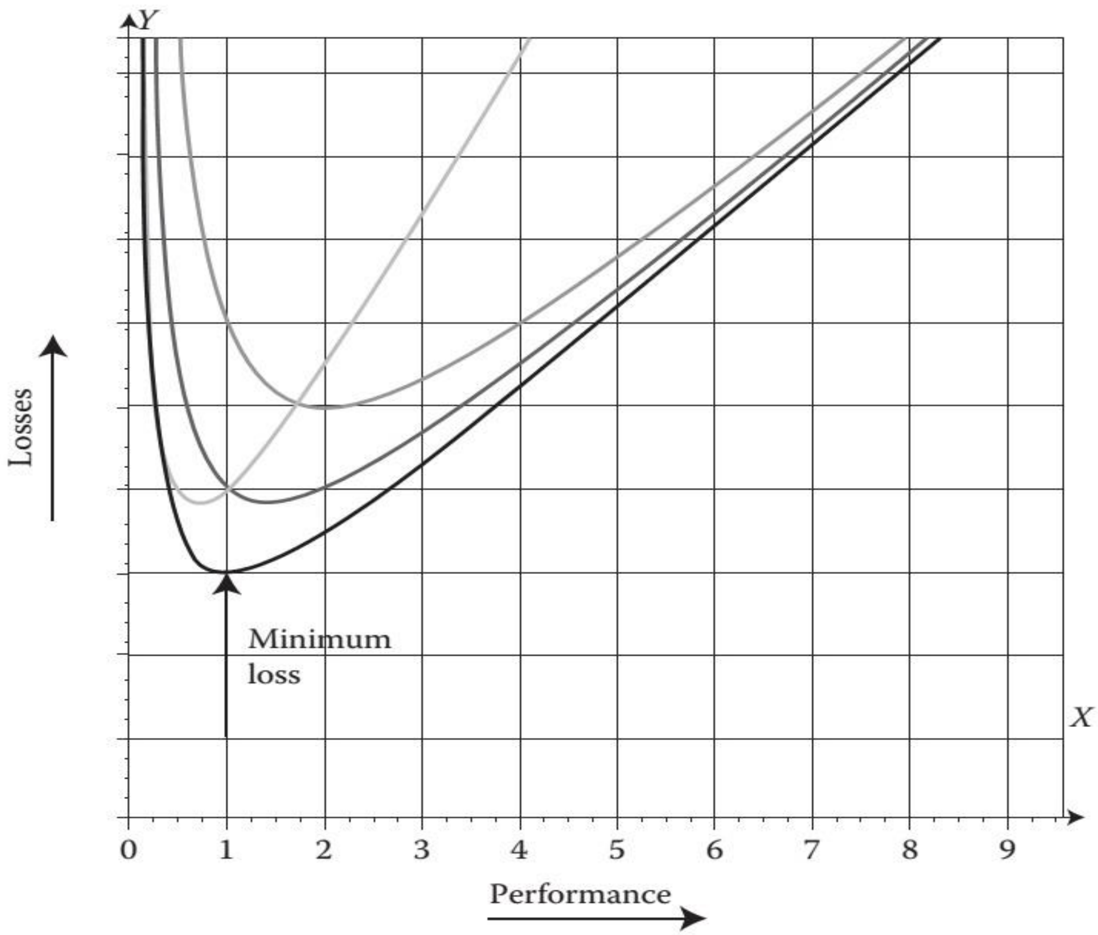

Figure 8. Pareto-efficient Quality Loss Function Optimized for Minimum Loss (from Langford 2012, 338)

C. GENERAL QUALITY LOSS FUNCTION

Given the benefits to program managers in decision-making using the minimum quality loss function, it is beneficial to find a general quality loss

function. Next, a derivation of the general quality loss function is shown, based off Choi and Langford work (2008).

1. Assumptions

According to Langford, there are seven assumptions made to develop a general quality loss function:

1. The total quality loss ($L_n(x)$) consists of the stakeholders' loss plus unknown losses.
2. If the level of quality equals the target value of the quality (i.e., *m*), the total quality loss is to be zero (or the minimum loss that is inherent in the system).
3. If the acquisition phase is production and deployment, the value of shape parameter *n* is equal to 2.
4. The minimum value of a shape parameter is close to zero and the value of the shape parameter in the concept refinement phase of the acquisition phases varies from 0 to 1.
5. When the acquisitions phases are the technology development or system development and demonstration phase, the range value of shape parameter varies from greater than one to less than two.
6. After the production and deployment phase, the value of the shape parameter is greater than two.
7. The probability distribution of the quality response remains the same regardless of the acquisition phases. (2012)

2. Notation

As per Langford (2012):

C_b : Baseline cost with a constant value.

C_s : If the type of quality characteristic is smaller-the-better, this means a proportionality constant of stakeholder's loss per response of quality. Additionally, if the type of quality characteristic is larger-the-better, it means a proportionality constant of developer's or manufacturer's loss per response of quality.

C_1 : If the type of quality characteristic is larger-the-better, this means a proportionality constant of developer's or manufacturer's loss per response of quality. Additionally, if the type of quality

characteristic is smaller-the-better, it means proportionality constant of the stakeholder's loss per response of quality.

n : Shape parameter for representing an acquisition phase of a weapon system ($n > 0$).

x : Response of quality.

$L_n(x)$: Total quality loss per piece in the case of shape parameter *n* and quality response *x*.

L_n : Expected quality loss per piece in the case of shape parameter *n* and quality response *x*.(2012)

3. Derivation of General Quality Loss

Given the assumption 1 and equation 4, 5, and 6, a "general quality loss function can be described as the following Equation" 7 (Langford 2012). Equation 7 "covers all quality characteristics such as nominal-the-best, smaller-the-better, and larger-the-better" (Langford).

$$L_n(x) = C_b + C_s x^n + C_1 x^{-n} \tag{7}$$

After applying the assumption 2 into Equation 7, one can get equations 8 and 9 as follows. If the response of quality equals to the target value *m*, the total quality loss is zero (Equation 8) and the result of differentiation for the response of quality having the target value is also to be zero as in Equation 9.

$$L_n(m) = C_b + C_s m^n + C_1 m^{-n} = 0 \tag{8}$$

$$L_n^{'}(m) = nC_s x^{n-1} - nC_1 m^{-n-1} = 0 \tag{9}$$

If one incorporates the specific value of *n* into Equations 8 and 9, one obtains the general loss function as follows. If the value of *n* equals to 1, one obtains the following results:

$$L_1(m) = C_b + C_s m^1 + C_1 m^{-1} = 0 \tag{10}$$

$$\mathrm{L}_1^{'}(m) = C_s m^0 - C_1 m^{-2} = 0 \tag{11}$$

After solving Equations 10 and 11, one obtains the following:

$$C_1 = C_s m^2, C_b = -2C_s m$$

If *n* equals to 2, one obtains the following results:

$$L_2(m) = C_b + C_s m^2 + C_1 m^{-2} = 0 \tag{12}$$

$$\mathrm{L}_2^{'}(m) = 2C_s m - 2C_1 m^{-3} = 0 \tag{13}$$

After solving Equations 12 and 13, one obtains the following results:

$$C_1 = C_s m^4, C_b = -2C_s m^2$$

After iterating in the above manner, one generate a quality loss function as shown in Table 2.

Table 2. Results of Iterative Process for Generating a Quality Loss Function (from Choi and Langford 2008, 36)

Results of Iterative Process for Generating a Quality Loss Function

n	C_1	C_b	$L_n(x)$
1	$C_1 = C_s m^2$	$C_b = -2C_s m^1$	$L_1(x) = -2C_s m^1 + C_s x^1 + C_s m^{2\times1} x^{-1}$
2	$C_1 = C_s m^4$	$C_b = -2C_s m^2$	$L_2(x) = -2C_s m^2 + C_s x^2 + C_s m^{2\times2} x^{-2}$
3	$C_1 = C_s m^6$	$C_b = -2C_s m^3$	$L_3(x) = -2C_s m^3 + C_s x^3 + C_s m^{2\times3} x^{-3}$
4	$C_1 = C_s m^8$	$C_b = -2C_s m^4$	$L_4(x) = -2C_s m^4 + C_s x^4 + C_s m^{2\times4} x^{-4}$
n	$C_1 = C_s m^{2n}$	$C_b = -2C_s m^n$	$L_n(x) = -2C_s m^{2n} + C_s x^n + C_s m^{2n} x^{-n}$

As shown in the last row of Table 2, a general quality loss function is presented, detailed as follows:

$$\begin{aligned} L_n(x) &= -2C_s m^n + C_s x^n + C_s m^{2n} x^{(-n)} \\ &= -2C_s m^n + C_s x^n (1 + m^{2n} x^{(-2n)}) \end{aligned} \tag{14}$$

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V. ANALYSIS OF SATELLITE SCHEDULING LOSSES

A. APPLICATION TO SATELLITE

Now that a satellite optimization algorithm has been established and a general quality loss function determined, the next step is to investigate and answer the question "What are the expected losses between optimizing for quicker response times versus maximum number of image takes?" Losses can be thought of in terms of EMMI, but for the purposes of this analysis, the focus will be on material wealth (American dollars).

As stated in Chapter III, there are three different broad groups of stakeholder's requirements when in relation to response time of image takes. The following notations will be used:

- Group A: Those stakeholders who need images within less than 48 hours of the request. A target value of 12 hours will be assumed for this group. Example stakeholders are warfare, forest fires, and severe weather tracking.
- Group B: Those stakeholders who need images no later than three weeks from image take request. A target value of 14 days will be assumed for this group. Example stakeholders those trying to track tidal action, algal blossoms, ice flows, fisheries, and agriculture.
- Group C: Those stakeholders who need images on a bi-annual more less basis. A target value for of six months is set for this operation. Examples stakeholders with these requirements might be those performing cartography, regional planning, and landscape monitoring.

B. SATELLITE IMAGERY COSTS

Next the cost of satellite imagery must be established, to be able to get a baseline for the monetary losses. There are not many retailers of commercial satellite imagery available in today's market, with Digital Globe being arguably the most dominate with its fleet of five satellites capable of collecting images. Digital Globe and other satellite owners distribute their images through multiple distribution companies. One of them, Apollo Mapping, can process orders for images from nine different satellites and one terrestrial plane. The available

satellite images are Worldview-1, Worldview-2, Pleides 1A/1B, GeoEye-1, Quickbird, KOMPSAT-3, EROS B, and IKONOS (Apollo Mapping 2014).

The prices of the images available from Apollo Mapping varies greatly depending on the how new the image is and if the user had to specifically request it or not. Via a direct phone call with the Apollo Mapping, the pricing estimates were provided in Table 3.

Table 3. Apollo Mapping Pricing Estimates

Time Period	Cost ($/km^2)
Less than 1 week	$150
3 week	$43
6 months	$20

C. DETERMINING LOSS FUNCTIONS

Given the above information and loss functions, four distinct loss functions are generated and will be used to show application in the decision-making process for program managers trying to schedule optimally a satellite constellation.

1. Loss Function for Time Required to Take Image: Group A

The first loss function to determine is for the time required to fulfill an image request for stakeholders in Group A. Group A, as defined above requires that images be fulfilled in 24-48 hours in order to be of worth to their application. Looking at the three different types of loss functions from Chapter IV, it is easily seen that the time required to fulfill an image take is a smaller-the-better type characteristic. Therefore, the loss function is defined by Equation 5 above, which is restated below:

$$L = ky^2 \quad k = \frac{A_0}{{y_0}^2}$$

where L is the loss ($), k is the proportionality constant ($\frac{\$}{hr^2}$), y is the output value (hr.), A_0 is the consumer loss ($), and y_0 is the maximum tolerated output value (hr.).

In order to use this equation, the proportionality constant, *k*, must be established. First, A_0 is defined as the consumer loss. This loss will greatly depend on the eventual stakeholder who needs the data. For illustrative purposes, the stakeholder will be assumed to be a military organization planning an operation in need of very near term satellite imagery. Due to the transient nature of the proposed enemy, the stakeholder determines that their images should be less than 24 hours old and anything older than 48 hours is not useable for their purposes. The actual value of this data, on which the entire operation hinges on is determined to be $100,000. That is approximately the cost the operation would be if the operation were carried out with no loss of life or equipment. Therefore, A_0 for this example is set at $100,000. Next, y_0 is determined by the stakeholder. Since maximum tolerated time for the image request to be returned is 48 hours, that is y_0. Therefore:

$$k = \frac{A_0}{{y_0}^2} = \frac{\$100,000}{(48hr)^2} = 43\frac{\$}{hr^2}$$

Applying *k* into the loss function, we get:

$$L = ky^2 = 43\frac{\$}{hr^2}y^2hr^2 = \$43y^2$$

Therefore, the loss function for image take time for Group A is shown in Figure 9.

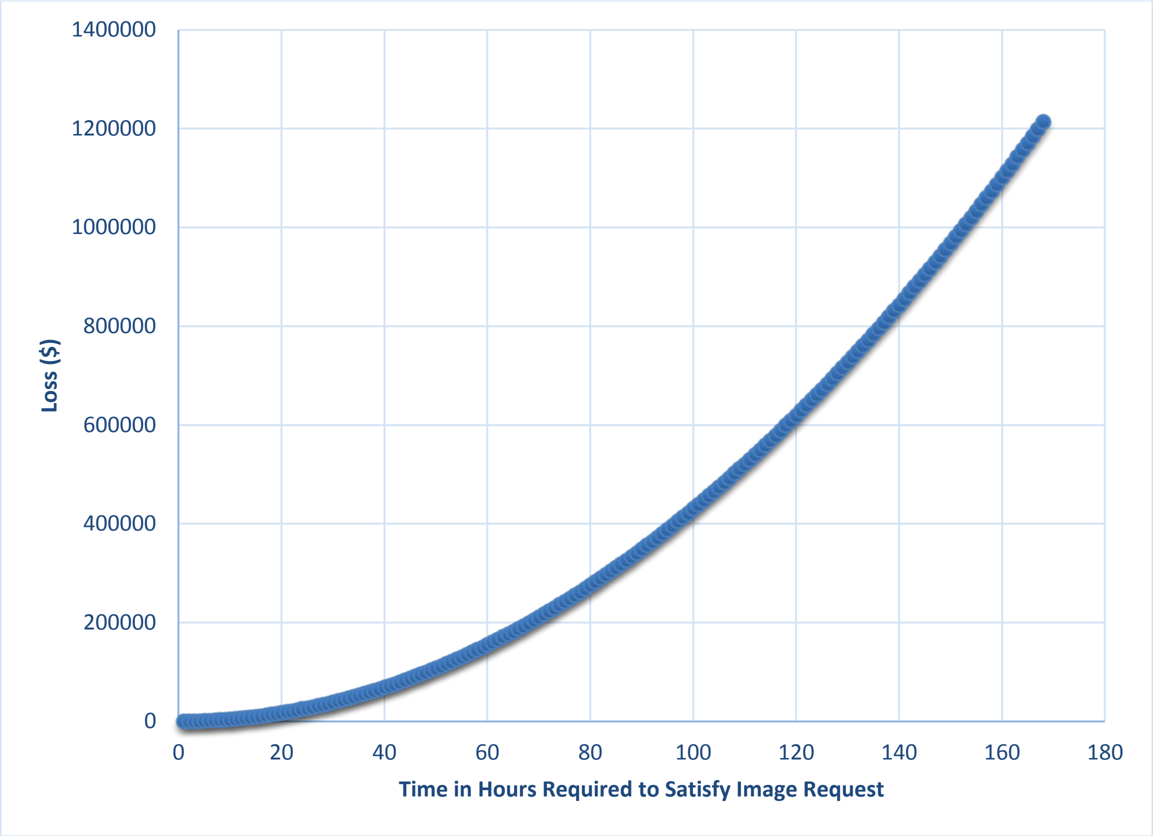

Figure 9. Loss Due to Quickness of Response: Group A

2. Loss Function for Time Required to Take Image: Group B

The second loss function to determine is for the time required to fulfill an image request for stakeholders in Group B. Group B, as defined above requires that images be fulfilled in two to three weeks in order to be of value to their application. Again, this loss function is represented by a smaller-the-better, which is defined as:

$$L = ky^2 \quad k = \frac{A_0}{{y_0}^2}$$

where L is the loss ($), k is the proportionality constant ($\frac{\$}{days^2}$), y is the output value (days), A_0 is the consumer loss ($), and y_0 is the maximum tolerated output value (days).

The proportionality constant, *k*, must be again be established. In this case, A_0 will be different than in case A. For stakeholders in Group B, the consequences and therefore the loss for late data is logically lower. For illustrative purposes, if the stakeholder is assumed to be a scientific organization tracking tidal conditions. Missing images or getting images a few days late will not carry as much weight as those items in group B. Since these numbers again depend solely on the exact nature of the program and the number of people working and expected loss of man hours given missing data, for this example numbers will be set. It will be assumed that for week of delayed data, it costs a research company $50,000 of wasted time not being able to continue their primary research initiative. They need data within a three-week period; otherwise, they will miss their primary deadline for their report. Therefore, A_0 for this example is set at $50,000. Next, y_0 is determined by the stakeholder. Since maximum tolerated time for the image request to be returned is three weeks, that is y_0. Therefore:

$$k = \frac{A_0}{{y_0}^2} = \frac{\$50,000}{(21day)^2} = 113\frac{\$}{day^2}$$

Applying *k* into the loss function, we get:

$$L = ky^2 = 113\frac{\$}{day^2}y^2day^2 = \$113y^2$$

Therefore, the loss function for image take time for Group B is shown in Figure 10.

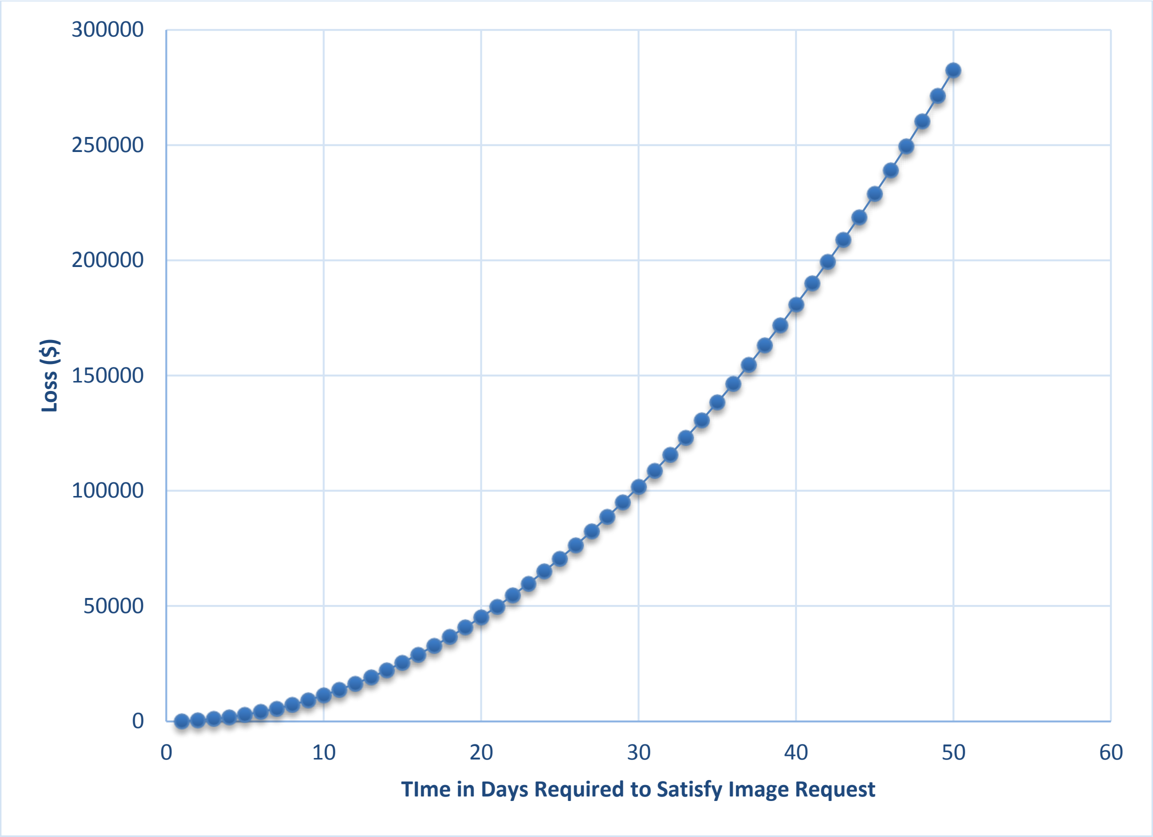

Figure 10. Loss Due to Quickness of Response: Group B

3. Loss Function for Time Required to Take Image: Group C

Similarly, for Group C, the loss function for the time required to satisfy image requests is a smaller-the-better function, which is:

$$L = ky^2 \quad k = \frac{A_0}{{y_0}^2}$$

where L is the loss ($), k is the proportionality constant ($\frac{\$}{weeks^2}$), y is the output value (weeks), A_0 is the consumer loss ($), and y_0 is the maximum tolerated output value (weeks).

The proportionality constant, *k*, must be again be established. A_0 will again change be different than case A and B. For stakeholders in Group C, the consequences and therefore the loss for late data is logically lower. For

illustrative purposes, if the stakeholder is assumed to be a scientific organization making maps. Missing images or getting images a few weeks late will not carry as much weight as those items in group B or A. Since these numbers again depend solely on the exact nature of the program and the number of people working and expected loss of man hours given missing data, for this example numbers will be set. It will be assumed that for each week of delayed data past months, it costs a research company $10,000 of wasted time not being able to continue their mapping operations. They need data within a six-month period, otherwise the risk of new development and outdated maps is too high. Therefore, A_0 for this example is set at $10,000. Next, y_0 is determined by the stakeholder. Since maximum tolerated time for the image request to be returned is 26 weeks, that is y_0. Therefore:

$$k = \frac{A_0}{{y_0}^2} = \frac{\$10,000}{(26 weeks)^2} = 15 \frac{\$}{week^2}$$

Applying *k* into the loss function, we get:

$$L = ky^2 = 15 \frac{\$}{week^2} y^2 week^2 = \$15 y^2$$

Therefore, the loss function for image response time for Group B is shown in Figure 11.

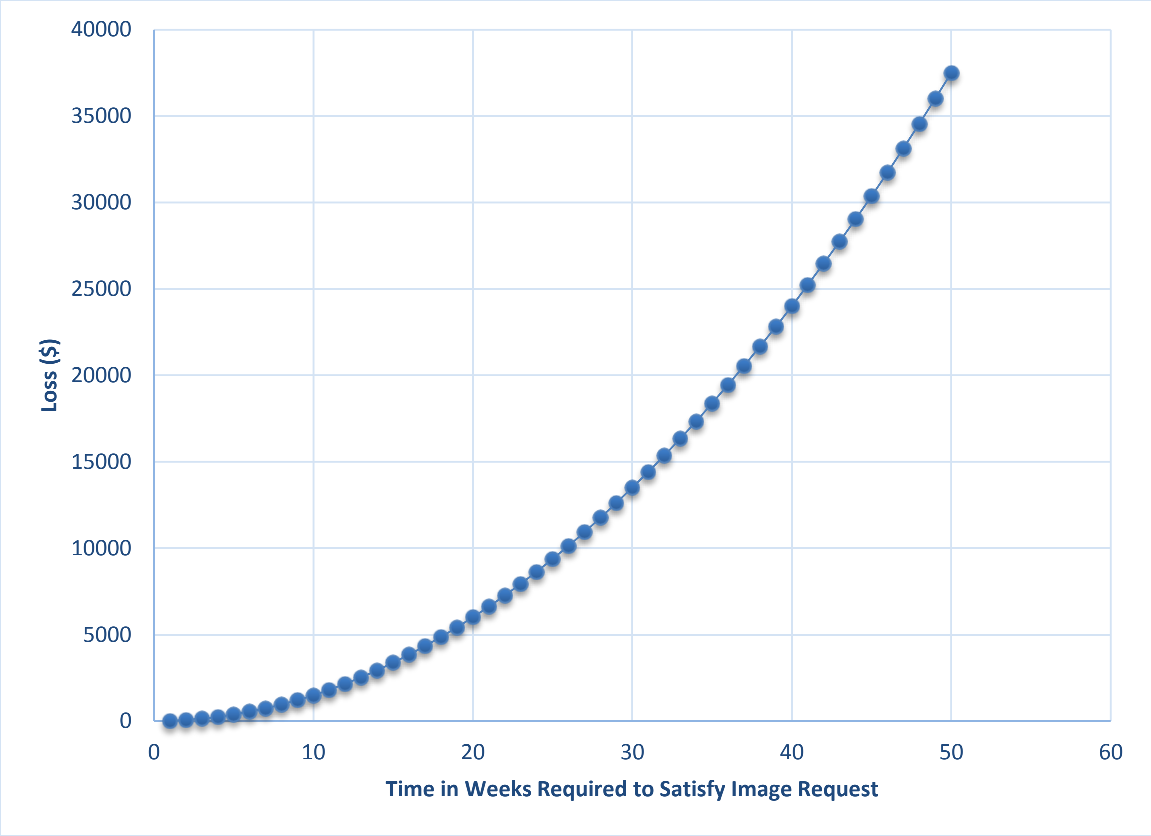

Figure 11. Loss Due to Quickness of Response: Group C

4. Loss Function for Number of Image Request Satisfied

Lastly, the loss function for the number of satisfied image requests must be established. For the owner of the satellites, the number of image requests satisfied relates directly to how much profit they make; therefore, it is a larger-the-better function. From Equation 6, larger-the-better loss function is represented by the following function:

$$L = k\frac{1}{y^2} \quad k = A_0 y_0^{\ 2}$$

where L is the loss ($), k is the proportionality constant ($\$ \bullet \text{Number images}^2$), y is the output value (number images), A_0 is the consumer loss ($), and y_0 is the minimum tolerated output value (number images).

In order to use this function, the proportionality constant must again be defined. Establishing A_0 from the companies perspective is a challenging order. Due to the nature of the commercial business, getting their operating costs or internal limits on the number of satisfied images captured each day is challenging. Therefore some basic assumptions are made to set A_0. First, this exercise will assume one image is 1 km^2 in size. Next, according to Digital Globe, we see that Worldview-2, Digital Globe’s newest satellite, is capable of taking images of 1,000,000 km^2 area per day, which means they can accomplish about 1,000,000 images per day. The second assumption required it how much a distributor like Apollo Mapping is willing to lose in daily capacity to meet special user demands. This research will assume it is willing to only operate at half capacity on any given day, or to take 500,000 images in a day. Also, noting that the cost of an image varies greatly depending if a customer is demanding it on a short time scale, we will assume that Apollo Mapping projects its costs at the highest cost scale. Therefore, A_0 is equivalent to $150 per image. y_0 as defined above is set at 500,000 images per day minimum capacity. Therefore, k is shown to be:

$$k = A_0 y^2 = \$150(500,000 images)^2 = 3.75x10^{13}\$\bullet\text{number images}^2$$

Applying k into the loss function we get:

$$L = k\frac{1}{y^2} = 3.75x10^{13}\$\bullet\text{number images}^2 \frac{1}{y^2\text{number images}^2} = \frac{\$3.75x10^{13}}{y^2}$$

Therefore, the loss function for the number of images is shown in Figure 12.

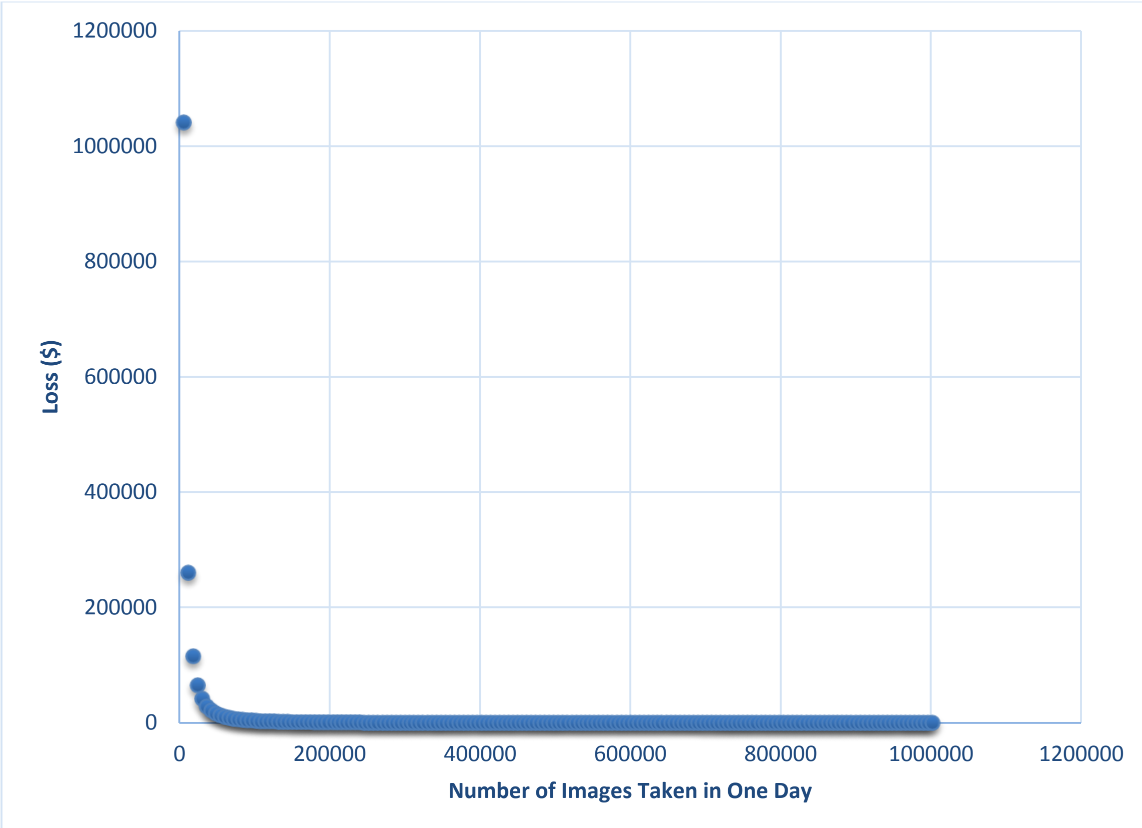

Figure 12. Loss Function for the number of Images per Day

D. DETERMINING MINIMUM QUALITY LOSS

After the individual loss functions are generated, the next step is to combine the loss functions, by adding them together, to get the optimal loss curves (Langford 2012). Once the curves are generated, the minimum dollar loss can quickly be found at the minimum of the curve. Since there are three different periods to investigate, there are three distinct loss curves that are generated below in Figures 13, 14, and 15.

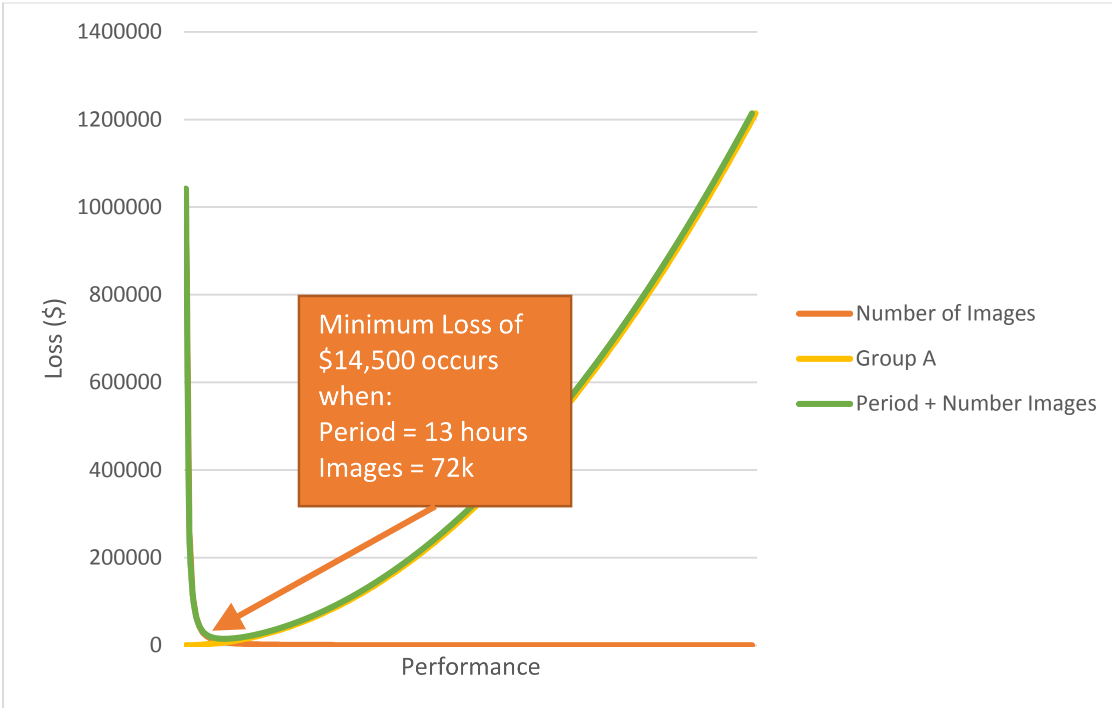

Figure 13. Group A Minimum Loss

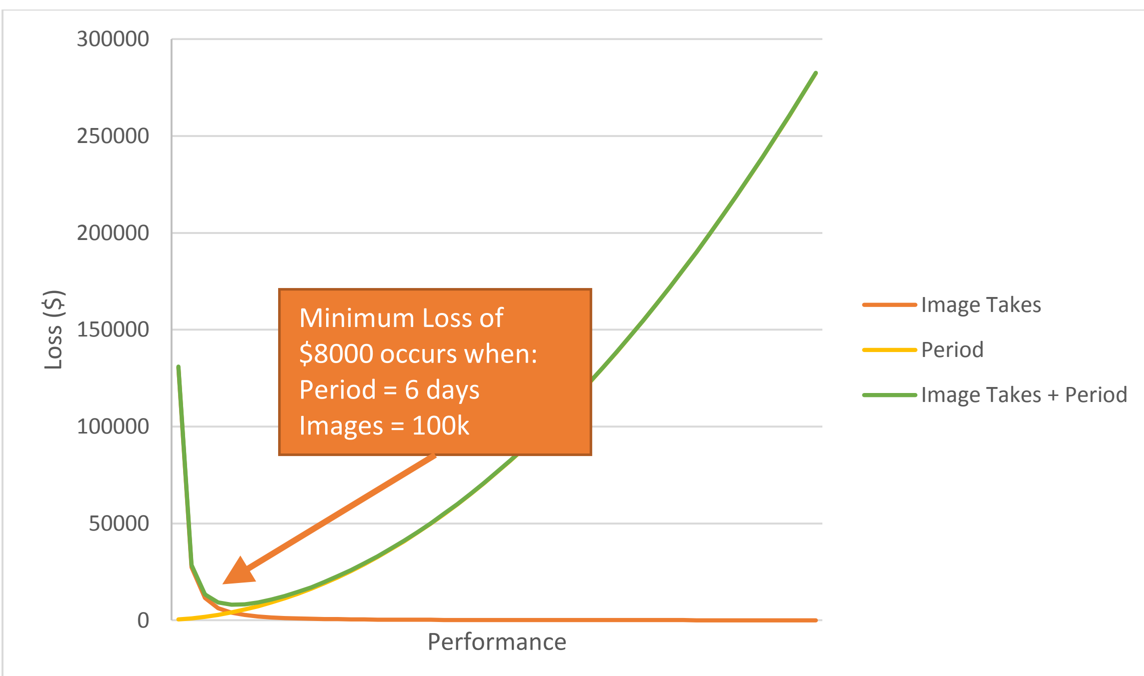

Figure 14. Group B Minimum Loss

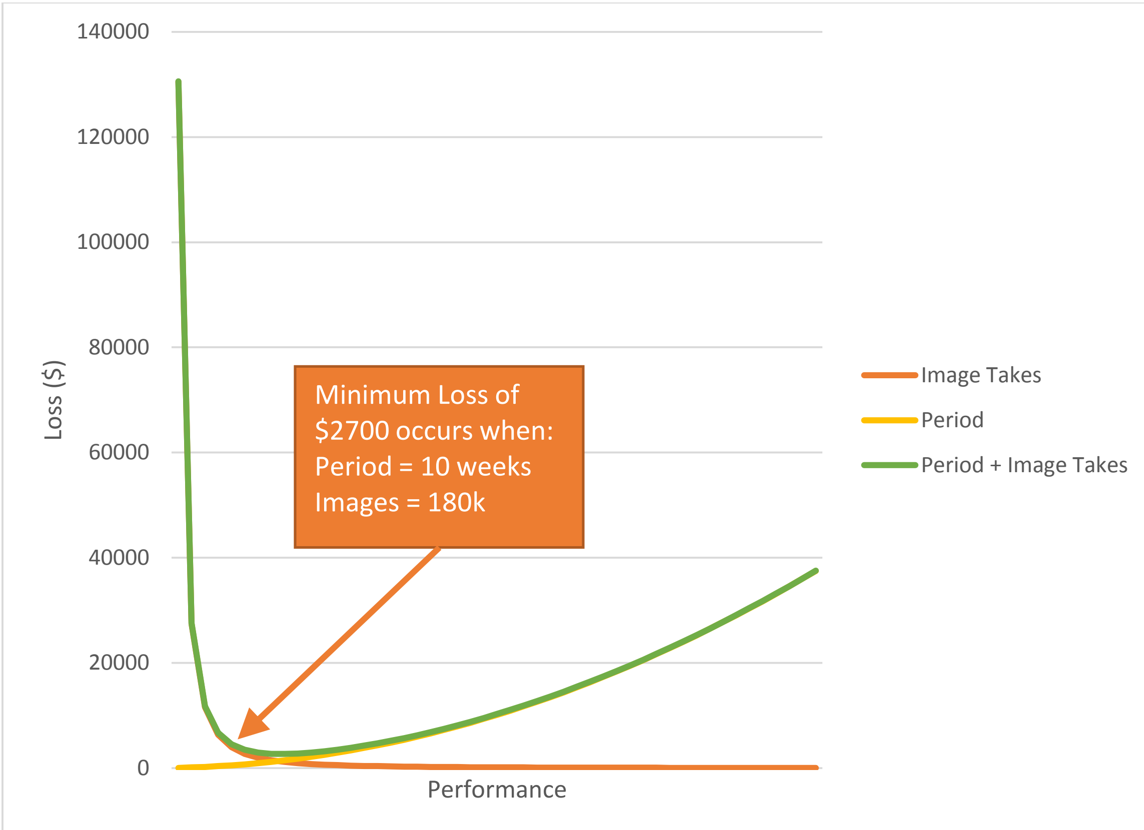

Figure 15. Group C Minimum Loss

Analysis of Figures 13, 14, and 15 provides insight into the optimization of the satellite scheduling problem. In Figure 13, Group A stakeholders (response time less than 48 hours) minimum loss function is determined to be $14,500. At that point in the negotiations, both parties are equally receiving losses and will be satisfied with the result of the negotiations. The values for the minimum loss for Group A is an average system response time of 13 hours and average number of images to be 72,000. In Figure 14, Group B stakeholders (response time less than three weeks) minimum loss function is determined to be $8,000. At that point in the negotiations, both parties are equally receiving losses and will be satisfied with the result of the negotiations. The values for the minimum loss for Group B is an average system response time of six days and average number of images to be 100,000. In Figure 15, Group C stakeholders (response time less than six months) minimum loss function is determined to be $2,700. At that point

in the negotiations, both parties are equally receiving losses and will be satisfied with the result of the negotiations. The values for the minimum loss for Group B is an average system response time of 10 weeks and average number of images to be 180,000.

For items of higher priority in which the images themselves cost more money, the expected minimum loss is higher. Given that the response time is so much shorter and fewer images in total are taken that general trend seems logical. That pattern, however, does not agree with De Floria's conclusion that the *m* value for optimizing for both period and the number of image takes are akin (2006). Since the time required for the system to respond is a smaller-the-better loss function and the number of data takes is a larger-the-better response, those two factors are in opposition to each other. It is only when the minimum loss for the system as a whole is modeled together that a weighting factor can be places on the optimization of the algorithm. Also, since loss functions are established, it is possible to evaluate different optimization algorithms against these two factors to determine if the results are providing a minimum loss to all stakeholders.

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VI. CONCLUSIONS

A satellite scheduling algorithm displaying tension between two different factors was explored. The general quality loss function was developed, proven and four distinct loss functions were developed to investigate the performance of a satellite optimization algorithm. The utility of the loss functions is that it puts the abstract ideas of user response time and number of images taken into common units of dollars. The individual loss functions were combined to show at which point the minimum total system loss is found. This information can be used to inform the adjustment of the scheduling algorithm for specific stakeholder use cases. The methodology of using loss functions for the purpose of helping decision-making in the systems engineering process was investigated and shown to have some utility.

A. FUTURE WORK

This research can be continued on in two primary methods. First, there is a need to find historical data such that the analysis of the developed loss function can be proved. Secondly, more precise knowledge of a specific real world user case could be established so that a loss function could be developed for that application.

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APPENDIX. QUATERNIONS

October 16, 1843 William Rowan Hamilton, an Irish mathematician, discovered a set of equations used to describe a new algebra, called quaternions or Hamilton numbers. Hamilton, for the decade before his discovery, was trying to create the theory of triplets, which he thought would be useful in describing rotations in three-dimensional space, similar to how complex numbers can be used to represent rotations in two-dimensional space. However, on that fateful day in 1843, Hamilton, while walking with his wife Lady Hamilton along the Royal Canal in Dublin realized that he would need three complex numbers as part of his algebra, not just two. He was so excited by his discovery he carved the equation $i^2 = j^2 = k^2 = ijk = -1$ onto the nearby Broome Bridge (Buchmann 2011).

Quaternions, at their most basic level, can be used to display tension between forces. Satellite motion around the earth is just such a problem that quaternions can be employed. The basic two-body equation describes the forces between two masses as their gravities interact with each other. If there were no other forces acting two bodies in relative motion, then Equation A.1 would describe their motion with the following nonlinear second order differential equation:

$$\mathbf{r}'' = -\frac{\mu}{r^3}\mathbf{r} \text{ where } \mu = G(m_1 + m_2) \tag{A.1}$$

where r is the position vector, G is the center of gravity between the two bodies, and *m* represents the masses of the bodies (Curtis 2011). What follows is an introduction to quaternion algebra followed by a derivation using quaternions of the two-body equation completed by Jorg Waldvogel (Waldvogel 2007).

A. BASIC QUATERNION ALGEBRA

Quaternions consist of eight elements $\pm 1, \pm i, \pm j, \pm k$ satisfying the relations,

$i^2 = j^2 = k^2 = ijk = -1$ or, more explicitly

$\mathrm{i}^2 = j^2 = k^2 = -1$

$ij = -ji = k$

$jk = -kj = i$

$ki = -ik = j$

1 is the unit element. In order to work with quaternions, some basic quaternion algebra must be established. Quaternions were originally conceived with the above definitions and assuming that $\mathbf{x} = x_0 + x_1 i + x_2 j + x_3 k$ where $\mathrm{x}_0,...,\mathrm{x}_4$ are real numbers (throughout the following, bold characters will denote quaternions). Hamilton also noticed that imaginary components naturally group together and he called this the vector portion while the real part, u_0, is called a scalar. So, quaternions can also be written in the form $[\mathbf{x}, \mathrm{x}_0]$, with $\mathbf{x} = (\mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3)$. Real numbers, s, will be identified with quaternions $[0, s]$, and vectors $\mathbf{x} \in \mathbb{Q}$ with quaternions $[\mathbf{x}, 0]$ where $\mathbb{Q}$ denotes the set of all quaternions.

Quaternion multiplication is generally non-commutative, however, any quaternion commutes with a real so that:

$$c\mathbf{x} = \mathbf{x}c, c \in \mathbb{R}, \mathbf{x} \in \mathbb{Q} \tag{A.2}$$

For any three quaternions $\mathbf{x}, \mathbf{y}, \mathbf{z} \in \mathbb{Q}$ the associative law holds so that:

$$(\mathbf{xy})\mathbf{z} = \mathbf{x}(\mathbf{yz}) \tag{A.3}$$

The quaternion **u** may naturally be associated with the corresponding vector $\mathbf{x} = (\mathrm{x}_0, \mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3) \in \mathbb{R}^4$. For later convenience two particular quaternions are introduced: (1) the pure quaternion $\mathbf{x} = ix_1 + jx_2 + kx_3$ is associated with $\vec{x} = (\mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3) \in \mathbb{R}^3$, and (2) $\underset{\rightarrow}{x} = (\mathrm{x}_0, \mathrm{x}_1, \mathrm{x}_2)$ for the vector associated with the quaternion without *k*-component, $\mathbf{x} = x_0 + ix_1 + jx_2$.

The conjugate of the quaternion **u** is defined as Equation A.4.

$$\bar{\mathbf{x}} = x_o - ix_1 - jx_2 - kx_3 \tag{A.4}$$

Then the modulus $|\mathbf{x}|$ of **x** is obtained from:

$$|\mathbf{x}|^2 = \mathbf{x}\bar{\mathbf{x}} = \bar{\mathbf{x}}\mathbf{x} = \sum_{l=0}^{3} x_l^{\,2} \tag{A.5}$$

As transposition of a product of matrices, conjugation of a quaternion product reverses the order of its factors as shown in Equation A.6.

$$\overline{\mathbf{xy}} = \bar{\mathbf{y}}\bar{\mathbf{x}} \tag{A.6}$$

The two different kinds of division by $\mathbf{x} \neq 0$ are carried out by left- or right-multiplication with the inverse $\mathbf{x}^{-1} = \dfrac{\bar{\mathbf{x}}}{\mathbf{x}\bar{\mathbf{x}}}$.

In addition to the basic algebra presented, one useful application of quaternions is representing rotation in $\mathbb{R}^3$. Let $\vec{v} = (\mathrm{v}_1, \mathrm{v}_2, \mathrm{v}_3) \in \mathbb{R}^3, |\vec{v}| = 1$ be a unit vector defining an oriented rotation axis, and let ω be a rotation angle. Define the unit quaternion as $\mathbf{r} := \cos\dfrac{\omega}{2} + (iv_1 + jv_2 + kv_3)\sin\dfrac{\omega}{2}$. Furthermore, let $\vec{u} \in \mathbb{R}^3$ be an arbitrary vector, and let $\mathbf{u} = iu_1 + ju_2 + ku_3$ be associated pure quaternion. Then the mapping $\mathbf{u} \mapsto \mathbf{v} = \mathbf{r}\mathbf{u}\mathbf{r}^{-1}$ describes the right-handed rotation of $\vec{u}$ about the axis $\vec{v}$ through the angle ω (since **r** is a unit quaternion we have $\mathbf{r}^{-1} = \bar{\mathbf{r}}$) (Waldvogel 2007).

B. QUATERNION TWO-BODY EQUATION

In order to show the quaternion two-body motion, Waldvogel uses the well-known Kustaanheimo-Stiefel or KS regularization (Kustaanheimo and Stiefel 1965). The first step of which is to represent the KS regularization as a quaternion.

1. KS Transformation Quaternion Representation

Therefore a new type of conjugate is introduced named the star conjugate and shown in Equation A.7. $\mathbf{x}^*$ is defined as the star conjugate of quaternion $\mathbf{x} = x_0 + ix_1 + jx_2 + kx_3$.

$$\mathbf{x}^* := x_0 + ix_1 - jx_2 - kx_3 \tag{A.7}$$

The star conjugate of **u** can be expressed in terms of the conjugate of **u** as $\mathbf{x}^* = k\bar{\mathbf{x}}k^{-1} = -k\bar{\mathbf{x}}k$. Therefore the following properties are presented and can be easily verified:

$$(\mathbf{x}^*)^* = \mathbf{x}, \left|\mathbf{x}^*\right|^2 = \left|\mathbf{x}\right|^2, (\mathbf{xy})^* = \mathbf{y}^*\mathbf{x}^*$$

Now to defined the KS regularization in terms of quaternions, consider the mapping:

$$\mathbf{x} \in \mathbb{Q} \mapsto \mathbf{u} = \mathbf{xx}^* \tag{A.8}$$

Star conjugation immediately yields $\mathbf{u}^* = (\mathbf{x}^*)^*\mathbf{x}^*$; and hence **u** is a quaternion of the form $\mathbf{u} = u_0 + iu_1 + ju_2$, which may be associated with the vector $\underset{\rightarrow}{u} = (\mathrm{u}_0, \mathrm{u}_1, \mathrm{u}_2) \in \mathbb{R}^3$. From $\mathbf{x} = x_0 + ix_1 + jx_2 + kx_3$ we obtain

$$\begin{aligned} u_0 &= x_0{}^2 - x_1{}^2 - x_2{}^2 - x_3{}^2 \\ u_1 &= 2(\mathrm{x}_0\, x_1 - x_2 x_3) \\ u_2 &= 2(x_0 x_2 + x_1 x_3) \end{aligned}$$

which is the KS transformation in its classical form. Therefore we have

Theorem 1: The KS transformation $x = (x_0, x_1, x_2, x_3) \in \mathbb{R}^4 \mapsto \underset{\rightarrow}{u} = (u_0, u_1, u_2) \in \mathbb{R}^3$ is given by the quaternion relation $\mathbf{u} = \mathbf{xx}^*$, where $\mathbf{x} = x_0 + ix_1 + jx_2 + kx_3$, $\mathbf{u} = u_0 + iu_1 + ju_2$, and $\mathbf{x}^*$ is defined by Equation A.7.

Corollary 1: The norms of the vectors $\underset{\rightarrow}{u}$ and x satisfy Equation A.9.

$$r := \left\|\underset{\rightarrow}{u}\right\| = \left\|x\right\|^2 = \mathbf{x}\bar{\mathbf{x}} \tag{A.9}$$

2. Differentiation

Since the two-body equation is a second order differential equation, differentiation of the Equation A.8 is required. Therefore consider the noncommutativity of the quaternion product, the differential of the Equation A.8 becomes:

$$d\mathbf{u} = d\mathbf{x} \cdot \mathbf{x}^* + \mathbf{x} \cdot d\mathbf{x}^* \tag{A.10}$$

If we consider "bilinear relation" $2(x_2dx_0 - x_2dx_1 + x_1dx_2 - x_0dx_3) = 0$ and take the derivative we obtain $\mathbf{x} \cdot d\mathbf{x}^* - d\mathbf{x} \cdot \mathbf{x}^* = 0$. Combing this results with Equation A.10 we arrive at Equation A.11.

$$d\mathbf{u} = 2\mathbf{x} \cdot d\mathbf{x}^* \tag{A.11}$$

3. Regularization

Lastly, the formal procedures for KS-regularizing the two-body equations is presented using four parameters $(x_0, x_1, x_2, x_3) =: x \in \mathbb{R}^4$ and quaternion notation, $\mathbf{x} = x_0 + ix_1 + jx_2 + kx_3$. The planar case is the particular case $x_2 = x_3 = 0,$ i.e. $\mathbf{x} = x_0 + ix_1 \in \mathbb{Q}$. To begin, the differential equations of the two-body problem are presented in quaternion form in Equation A.12.

$$\mathbf{u}'' + \mu \frac{\mathbf{u}}{r^3} = 0 \in \mathbb{Q}, r = |\mathbf{u}|, (') = \frac{d}{dt} \tag{A.12}$$

where *t* is time, $\underset{\rightarrow}{u} = (\mathrm{u}_0, \mathrm{u}_1, \mathrm{u}_2) \in \mathbb{R}^3$ is the position of the moving particle, and $\mathbf{u} = u_0 + iu_1 + ju_2 \in \mathbb{Q}$ is the corresponding quaternion, and μ is the gravitation parameter.

Also, the energy integral of Equation A.12 is considered in Equation A.13.

$$\frac{1}{2}|\mathbf{u}'|^2 - \frac{\mu}{r} = -h = const \tag{A.13}$$

where the right-hand side, -*h*, has been chosen such that $h > 0$ corresponds to an elliptic orbit.

Next a three step approach is used to complete the differentiation.

a. Step 1: Slow-Motion Movie

A new fictitious time variable, τ, is created such that

$$dt = r \cdot d\tau, \frac{d}{d\tau}(\ ) = (\ )' \tag{A.14}$$

Therefore the ration $\frac{dt}{d\tau}$ of the two infinitesimal increments is made proportional to the distance *r*. The movie is run in slow-motion whenever *r* is small. Equation A.12 and A.13 are therefore transformed into Equation A.15 and A.16:

$$r\mathbf{u}'' - r'\mathbf{u}' + \mu\mathbf{u} = 0 \tag{A.15}$$

$$\frac{1}{2r^2}|\mathbf{u}'|^2 - \frac{\mu}{r} = -h \tag{A.16}$$

b. Step 2: Conformal Squaring with Quaternions

Step two of the regularization procedure consists of introducing new coordinates $\mathbf{x} \in \mathbb{Q}$ according to the KS mapping of Equation A.8 of a generalization of Levi-Civita's conformal squaring, which yields Equation A.17.

$$\mathbf{u} = \mathbf{x}\mathbf{x}^*,\ r := |\mathbf{u}| = |\mathbf{x}|^2 = \mathbf{x}\overline{\mathbf{x}} \tag{A.17}$$

Differentiation by means of Equation A.11 results with

$$\mathbf{u}' = 2\mathbf{x}\mathbf{x}^{*'},\ \mathbf{u}'' = 2\mathbf{x}\mathbf{x}^{*''} + 2\mathbf{x}'\mathbf{x}^*,\ r' = \mathbf{x}'\overline{\mathbf{x}} + \mathbf{x}\overline{\mathbf{x}}' \tag{A.18}$$

Substitution of Equation A.17 and A.18 into A.15 results in Equation A.19.

$$(\mathbf{x}\overline{\mathbf{x}})(2\mathbf{x}\mathbf{x}^{*''} + 2\mathbf{x}'\mathbf{x}^{*'}) - (\mathbf{x}'\overline{\mathbf{x}} + \mathbf{x}\overline{\mathbf{x}}')2\mathbf{x}\mathbf{x}^{*'} + \mu\mathbf{x}\mathbf{x}^* = 0 \tag{A.19}$$

Equation A.19 is simplified by observing that the second and third term compensate as shown in Equation A.20.

$$2(\mathbf{x}\overline{\mathbf{x}})\mathbf{x}'\mathbf{x}^{*'} - 2\mathbf{x}'(\mathbf{x}\overline{\mathbf{x}})\mathbf{x}^{*'} = 0 \tag{A.20}$$

Furthermore, by Equation A.2, A.3, and knowing that $\mathbf{x} \cdot d\mathbf{x}^* - d\mathbf{x} \cdot \mathbf{x}^* = 0$ the fourth term of Equation A.19 is simplified as shown in Equation A.21.

$$2(\mathbf{x}\overline{\mathbf{x}})(\mathbf{x}'\mathbf{x}^*) = -2\mathbf{x}(\overline{\mathbf{x}}'\mathbf{x}')\mathbf{x}^* = -2|\mathbf{x}'|^2\mathbf{x}\mathbf{x}^* \tag{A.21}$$

By using Equation A.21 and left-dividing by **u** Equation A.19 becomes Equation A.22.

$$2r\mathbf{x}^{*''} + (\mu - 2|\mathbf{x}'|^2)\mathbf{x}^* = 0 \tag{A.22}$$

c. *Step 3: Fixing the Energy*

From Equation A.5, A.11, and knowing that $(\mathbf{x}^*)^* = \mathbf{x}, |\mathbf{x}^*|^2 = |\mathbf{x}|^2, (\mathbf{x}\mathbf{y})^* = \mathbf{y}^*\mathbf{x}^*$ we have derive Equation A.23.

$$|\mathbf{u}'|^2 = \mathbf{u}'\overline{\mathbf{u}}' = 4\mathbf{x}(\mathbf{x}^{*'}\overline{\mathbf{x}}^{*'})\overline{\mathbf{x}} = 4r|\mathbf{x}'|^2 \tag{A.23}$$

Therefore Equation A.16 becomes Equation A.24.

$$\mu - 2|\mathbf{x}'|^2 = rh \tag{A.24}$$

Substituting Equation A.24 into the star conjugate of Equation A.22 and dividing by *r* finally yields Theorem 2.

Theorem 2: The KS transformation (Equation A.8) with the differential rule (Equation A.11) and the time transformation (Equation A.14) maps the spatial two-body problem (Equation A.12) into the quaternion differential equation shown in Equation A.25.

$$2\mathbf{x}'' + h\mathbf{x} = 0 \tag{A.25}$$

Equation A.25 describes the motion of four uncoupled harmonic oscillators with the common frequency $\omega := \sqrt{h/2}$.

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   Naval Postgraduate School
   Monterey, California